

For each function:

1. Identify any domain restrictions. At these restrictions, find the x value of any holes and /or the equations of any vertical asymptotes.
2. Use the concept of limits at infinity to determine whether there is a horizontal or slant asymptotes. Find the equation of this asymptote.

$$1. f(x) = \frac{x^3 - x^2 - 6x}{-3x^2 - 3x + 18}$$

$$\frac{x(x-3)(x+2)}{-3(x+3)(x-2)}$$

$$\begin{array}{r} -3x^2 - 3x + 18 \overline{) x^3 - x^2 - 6x + 0} \\ \underline{-x^3 + x^2 - 6x} \\ 2x^2 + 6x - 6 \\ \underline{-2x^2 + 6x - 6} \\ 12 \end{array}$$

Vertical asymptotes

$$x = -3$$

$$x = 2$$

slant asymptote

$$y = -\frac{1}{3}x + \frac{2}{3}$$

$$2. f(x) = \frac{x+4}{-2x-6}$$

$$\frac{x+4}{-2(x+3)}$$

vertical asymptote

$$x = -3$$

horizontal asymptote

$$y = -\frac{1}{2}$$

$$3. f(x) = \frac{x^3 - 16x}{-4x^2 + 4x + 24}$$

$$\frac{x(x-4)(x+4)}{-4(x-3)(x+2)}$$

$$\begin{array}{r} -4x^2 + 4x + 24 \overline{) x^3 + 0x^2 - 16x + 0} \\ \underline{-x^3 + 4x^2 - 16x} \\ 4x^2 + 22x + 24 \\ \underline{-4x^2 + 22x + 24} \\ 0 \end{array}$$

vertical asymptotes

$$x = 3 \quad x = -2$$

slant asymptote

$$y = -\frac{1}{4}x - \frac{1}{4}$$

$$4. f(x) = -\frac{4}{x^2 - 3x}$$

$$-\frac{4}{x(x-3)}$$

vertical asymptotes

$$x = 0$$

$$x = 3$$

horizontal asymptotes

$$y = 0$$