

Domain & Range

$$\sqrt{x^2+a}$$

↑
constant

Range could be a special case!

Find the domain and range of the given function.

1) $Y(t) = 3t^2 - 2t + 1$

D: \mathbb{R}

R: $[2/3, \infty)$

$3(t^2 - 2/3t + 1/9 - 1/9) + 1$

$3(t - 1/3)^2 - 1/3 + 1$

$3(t - 1/3)^2 + 2/3$

2) $g(z) = -z^2 - 4z + 7$

D: \mathbb{R}

R: $(-\infty, 11]$

$-(z^2 + 4z + 4 - 4) + 7$

$-(z+2)^2 + 4 + 7$

$-(z+2)^2 + 11$

* 3) $f(z) = 2 + \sqrt{z^2 + 1}$

D: \mathbb{R}

$z^2 + 1 \geq 0$

$z^2 \geq -1$

always true

R: $[3, \infty)$

Special case!

$2 + \sqrt{z^2 + 1}$

↑
always 0 or greater

$2 + \sqrt{\quad}$

4) $h(y) = -3\sqrt{14 + 3y}$

D: $[-14/3, \infty)$

$14 + 3y \geq 0$

$3y \geq -14$

$y \geq -14/3$

R: $(-\infty, 0]$

$-3\sqrt{14+3y}$

↑
always - always 0 or +

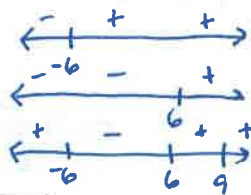
product is negative

Domain only 5)

$A(x) = \frac{4}{x-9} - \sqrt{x^2 - 36}$

↑
 $x-9=0$
 $x \neq 9$

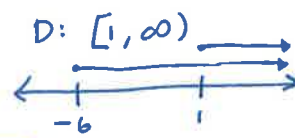
$x^2 - 36 \geq 0$
 $(x-6)(x+6) \geq 0$



D: $(-\infty, -6] \cup [6, 9) \cup (9, \infty)$

6) $f(z) = \sqrt{z-1} + \sqrt{z+6}$

$z-1 \geq 0$ $z+6 \geq 0$
 $z \geq 1$ and $z \geq -6$



R: $[0, \infty)$

$\sqrt{z-1} + \sqrt{z+6}$
0 or + 0 or +

7) $g(x) = \sqrt{10x - 15}$

D: $[3/2, \infty)$

R: $[0, \infty)$

$10x - 15 \geq 0$

$10x \geq 15$

$x \geq 3/2$

$\sqrt{10x-15}$

0 or +

Domain only 8)

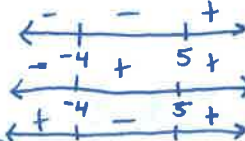
8) $h(z) = \sqrt{z^2 - z - 20}$

D: $(-\infty, -4] \cup [5, \infty)$

R: $[0, \infty)$

$z^2 - z - 20 \geq 0$

$(z-5)(z+4) \geq 0$



Domain only 9)

9) $M(x) = 5 - |x+8|$

D: \mathbb{R}

R: $(-\infty, 5]$

$5 - |x+8|$

always 0 or +
but you are subtracting
it

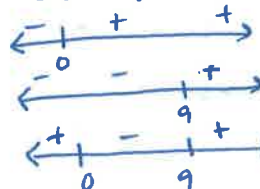
$A(z) = \sqrt{z^2 - 9z}$

D: $(-\infty, 0] \cup [9, \infty)$

R: $[0, \infty)$

$z^2 - 9z \geq 0$

$z(z-9) \geq 0$



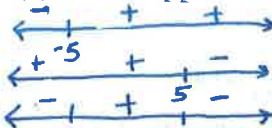
11) $f(x) = 9 - x$
linear

$D: \mathbb{R} \quad R: \mathbb{R}$

12) $g(x) = \sqrt{25 - x^2}$

$D: [-5, 5]$

$25 - x^2 \geq 0$
 $(5-x)(5+x) \geq 0$



Watch signs!

Special case!

$R: [0, 5]$

$\sqrt{25 - x^2}$

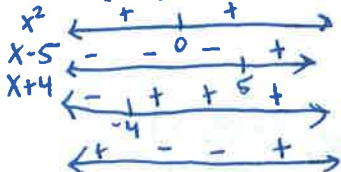
0 or +, but you're subtracting

Domain only

13) $h(x) = \sqrt{x^4 - x^3 - 20x^2}$

$D: (-\infty, -4] \cup [5, \infty) \quad R: [0, \infty)$

$x^2(x^2 - x - 20) \geq 0$



Find the domain of the given function.

15) $f(w) = \frac{w^3 - 3w + 1}{12w - 7}$

$D: (-\infty, 7/12) \cup (7/12, \infty)$

$12w - 7 = 0$

$12w = 7$

$w = 7/12$

16) $R(z) = \frac{5}{z^3 + 10z^2 + 9z}$

$z^3 + 10z^2 + 9z = 0$

$z(z^2 + 10z + 9) = 0$

$z(z+9)(z+1) = 0$

$z = 0 \quad z = -9 \quad z = -1$

D:

$(-\infty, -9) \cup (-9, -1) \cup (-1, 0) \cup (0, \infty)$

17) $g(t) = \frac{6t - t^3}{7 - t - 4t^2}$

$D: (-\infty, \frac{1+\sqrt{113}}{-8}) \cup (\frac{1-\sqrt{113}}{-8}, \infty)$

$7 - t - 4t^2 \geq 0$

$-4t^2 - t + 7 \geq 0$

$\frac{-1 \pm \sqrt{1 - 4(-4)(7)}}{2(-4)} = \frac{1 \pm \sqrt{113}}{-8}$

plug into calculator to see which one is larger

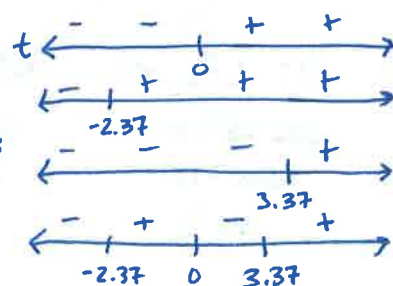
18) $P(t) = \frac{5t + 1}{\sqrt{t^3 - t^2 - 8t}}$

$D: (\frac{1-\sqrt{33}}{2}, 0) \cup (\frac{1+\sqrt{33}}{2}, \infty)$

$t^3 - t^2 - 8t > 0$

$t(t^2 - t - 8) > 0$

$\frac{1 \pm \sqrt{1 - 4(1)(-8)}}{2(1)} = \frac{1 \pm \sqrt{33}}{2}$

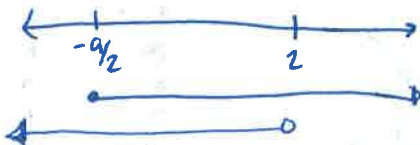


19) $h(y) = \sqrt{2y+9} - \frac{1}{\sqrt{2-y}}$ $D: [-9/2, 2)$

$2y+9 \geq 0$ AND $2-y > 0$

$2y \geq -9$ $-y > -2$

$y \geq -9/2$ $y < 2$



20) $Q(y) = \sqrt{y^2+1} - \sqrt[3]{1-y}$

$D: \mathbb{R}$

$y^2+1 \geq 0$ AND can be

$y^2 \geq -1$ always since it's cube root

\mathbb{R}

\mathbb{R}

$$21) h(x) = \frac{2+x}{8x-1}$$

$$8x-1=0$$

$$x = \frac{1}{8}$$

$$D: (-\infty, \frac{1}{8}) \cup (\frac{1}{8}, \infty)$$

$$22) A(t) = \frac{t^2-4}{t^2+6t-7}$$

$$t^2+6t-7=0$$

$$(t+7)(t-1)=0$$

$$t=-7 \quad t=1$$

$$D: (-\infty, -7) \cup (-7, 1) \cup (1, \infty)$$

$$23) h(w) = \frac{w^2+3w+2}{w^2+12w+36}$$

$$w^2+12w+36=0$$

$$(w+6)(w+6)=0$$

$$w=-6$$

$$D: (-\infty, -6) \cup (-6, \infty)$$

$$24) f(t) = \frac{10t}{\sqrt{6-4t}}$$

$$6-4t > 0$$

$$-4t > -6$$

$$t < \frac{3}{2}$$

$$D: (-\infty, \frac{3}{2})$$

$$25) f(w) = \frac{\sqrt{w+7}}{\sqrt{2-w}}$$

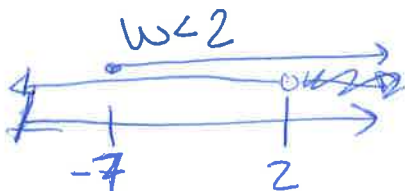
$$D: [-7, 2)$$

$$2-w > 0$$

$$-w > -2$$

$$w+7 \geq 0$$

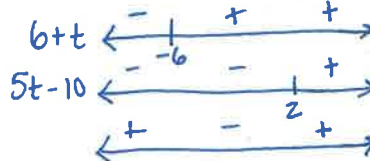
$$w \geq -7$$



$$26) g(t) = \sqrt{\frac{6+t}{5t-10}}$$

$$D: (-\infty, -6) \cup (2, \infty)$$

$$\text{same as } \frac{6+t}{5t-10} > 0$$



27) Create a function whose domain is all real numbers and range is $(-2, \infty)$.

Answers vary. Ex:

$$f(x) = -2 + |3x^2 - 1|$$

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3) $f(z) = 2 + \sqrt{z^2 + 1}$

4) $h(y) = -3\sqrt{14 + 3y}$

5) $A(x) = \frac{4}{x-9} - \sqrt{x^2 - 36}$

6) $f(z) = \sqrt{z-1} + \sqrt{z+6}$

7) $g(x) = \sqrt{10x - 15}$

8) $h(z) = \sqrt{z^2 - z - 20}$

9) $M(x) = 5 - |x + 8|$

10) $A(z) = \sqrt{z^2 - 9z}$

11) $f(x) = 9 - x$

12) $g(x) = \sqrt{25 - x^2}$

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14) $P(z) = z^2 - 4$

Find the domain of the given function.

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16) $R(z) = \frac{5}{z^3 + 10z^2 + 9z}$

17) $g(t) = \frac{6t - t^3}{7 - t - 4t^2}$

18) $P(t) = \frac{5t + 1}{\sqrt{t^3 - t^2 - 8t}}$

19) $h(y) = \sqrt{2y + 9} - \frac{1}{\sqrt{2 - y}}$

20) $Q(y) = \sqrt{y^2 + 1} - \sqrt[3]{1 - y}$

$$21) h(x) = \frac{2+x}{8x-1}$$

$$22) A(t) = \frac{t^2-4}{t^2+6t-7}$$

$$23) h(w) = \frac{w^2+3w+2}{w^2+12w+36}$$

$$24) f(t) = \frac{10t}{\sqrt{6-4t}}$$

$$25) f(w) = \frac{\sqrt{w+7}}{\sqrt{2-w}}$$

$$26) g(t) = \sqrt{\frac{6+t}{5t-10}}$$

27) Create a function whose domain is all real numbers and range is $(-2, \infty)$.

