Calculus

Name Key

Practice - Derivatives #1

Date

Use the definition of the derivative to find the derivative of each function with respect to *x*.

1) $f(x) = 4x - 4$

$\lim_{h\to 0} \left(\frac{1}{h}\right) 4(x+h) - 4 - (4x-4)$

$\lim_{h\to 0} \left(\frac{1}{h}\right) 4x+4h-4-4x+4$

$f'(x) = 4$

2) $f(x) = -4x^2 - 4x + 4$

$\lim_{h\to 0} \left(\frac{1}{h}\right) 4(x+h)^2 - 4(x+h) + 4 - (-4x^2 - 4x + 4)$

$\lim_{h\to 0} \left(\frac{1}{h}\right) 4(x^2+2hx+h^2) - 4x - 4h + 4 + 4x^2 + 4x - 4$

$\lim_{h\to 0} \left(\frac{1}{h}\right) -4x^2 - 8hx - 4h^2 - 4x - 4h + 4 + 4x^2 + 4x - 4$

$\lim_{h\to 0} \left(\frac{1}{h}\right) h(-8x - 4h - 4)$

$f'(x) = -8x - 4$

3) $f(x) = \sqrt{-3x - 3}$

$\lim_{h\to 0} \left(\frac{1}{h}\right) \sqrt{-3(x+h)-3} - \sqrt{-3x-3}$

$\lim_{h\to 0} \left(\frac{1}{h}\right) \left(\sqrt{-3x-3h-3} - \sqrt{-3x-3}\right) \cdot \frac{\left(\sqrt{-3x-3h-3} + \sqrt{-3x-3}\right)}{\left(\sqrt{-3x-3h-3} + \sqrt{-3x-3}\right)}$

$\lim_{h\to 0} \frac{-3x-3h-3+(3x+3)}{h\left(\sqrt{-3x-3h-3} + \sqrt{-3x-3}\right)}$

$f'(x) = \frac{-3}{2\sqrt{-3x-3}}$

4) $f(x) = -\frac{1}{x-1}$

$\lim_{h\to 0} \left(\frac{1}{h}\right) \left[\frac{-1}{x+h-1} - \left(\frac{-1}{x-1}\right)\right]$

$\lim_{h\to 0} \left(\frac{1}{h}\right) \left[\frac{-(x-1)}{(x+h-1)(x-1)} + \frac{x+h-1}{(x+h-1)(x-1)}\right]$

$\lim_{h\to 0} \frac{-x+1+x+h-1}{h(x+h-1)(x-1)}$

$f'(x) = \frac{1}{(x-1)^2}$

Differentiate each function with respect to *x*. Problems may contain constants a, b, and c.

5) $f(x) = x^5$ $f'(x) = 5x^4$

6) $f(x) = \frac{4}{x^3} = 4x^{-3}$ $f'(x) = -12x^{-4} = \frac{-12}{x^4}$

7) $f(x) = -5x^{-2}$ $f'(x) = 10x^{-3} = \frac{10}{x^3}$

8) $f(x) = -\frac{4}{5}x^3$ $f'(x) = \frac{-12x^2}{5}$

9) $f(x) = 4\sqrt[4]{x} = 4x^{1/4}$ $f'(x) = x^{-3/4} = \frac{1}{x^{3/4}}$ or $\frac{1}{\sqrt[4]{x^3}}$

10) $f(x) = \underbrace{-4a}_{\text{Constant}}x^{4b}$ $f'(x) = -16abx^{4b-1}$

11) $f(x) = \frac{2}{3}x^{3b}$ $f'(x) = 2bx^{3b-1}$

12) $f(x) = 3b\sqrt[4]{x}$ $3b\,x^{1/4}$ $f'(x) = \frac{3b}{4}x^{-3/4}$

$= \frac{3b}{4x^{3/4}}$ or $\frac{3b}{4\sqrt[4]{x^3}}$

Differentiate each function with respect to x.

13) $y = (2x^5 - 4)(-2x^2 - 1)$ $10x^4$ $-4x$

$\frac{dy}{dx} = (2x^5-4)(-4x) + (10x^4)(-2x^2-1)$

$= -8x^6 + 16x - 20x^6 - 10x^4$

$\frac{dy}{dx} = -28x^6 - 10x^4 + 16x$

14) $y = (4x^5 - 4)(-x^2 + 2)$ $20x^4$ $-2x$

$\frac{dy}{dx} = (4x^5-4)(-2x) + (20x^4)(-x^2+2)$

$-8x^6 + 8x - 20x^6 + 40x^4$

$\frac{dy}{dx} = -28x^6 + 40x^4 + 8x$

15) $y = (-2\sqrt[3]{x} - 4)(2x^4 - 5x^2 - 1)$ $-2x^{1/3}$ $-\frac{2}{3}x^{-2/3}$ $8x^3 - 10x$

$\frac{dy}{dx} = (-2x^{1/3}-4)(8x^3-10x) + (-\frac{2}{3}x^{-2/3})(2x^4-5x^2-1)$

$= -16x^{10/3} + 20x^{4/3} - 32x^3 + 40x - \frac{4}{3}x^{10/3} + \frac{10}{3}x^{4/3} + \frac{2}{3}x^{-2/3}$

$\frac{dy}{dx} = \frac{-52x^{10/3}}{3} - 32x^3 + \frac{70x^{4/3}}{3} + 40x + \frac{2}{3x^{2/3}}$

16) $y = \frac{5}{x^3 + 4}$ 0 $3x^2$

$\frac{dy}{dx} = \frac{0(x^3+4) - 5(3x^2)}{(x^3+4)^2} = \frac{-15x^2}{(x^3+4)^2}$

17) $y = \dfrac{3x^5 - 3x^2 + 2}{4x^2 - 3}$ $15x^4 - 6x$ / $8x$

$$\frac{dy}{dx} = \frac{(15x^4 - 6x)(4x^2 - 3) - (8x)(3x^5 - 3x^2 + 2)}{(4x^2 - 3)^2}$$

$$= \frac{60x^6 - 45x^4 - 24x^3 + 18x - 24x^6 + 24x^3 - 16x}{(4x^2 - 3)^2}$$

$$\frac{dy}{dx} = \frac{36x^6 - 45x^4 + 2x}{(4x^2 - 3)^2}$$

18) $y = \dfrac{5x^3 - 5}{4 + \frac{2}{x^3}}$ $15x^2$ / $-6x^{-4}$

$$\frac{dy}{dx} = \frac{15x^2(4 + 2x^{-3}) - (-6x^{-4})(5x^3 - 5)}{(4 + 2x^{-3})^2}$$

$$= \frac{60x^2 + 30x^{-1} + 30x^{-1} - 30x^{-4}}{(4 + 2x^{-3})^2}$$

$$\frac{dy}{dx} = \frac{60x^2 + 60x^{-1} - 30x^{-4}}{(4 + 2x^{-3})^2}$$

For each problem, find the instantaneous rate of change of the function at the given value.

19) $y = -x^2 + x - 1$; 2

$y' = -2x + 1$

$IRC = -2(2) + 1 = -3$

20) $y = -\dfrac{1}{x}$; 2 $\quad -1x^{-1}$

$y' = x^{-2}$

$IRC = 2^{-2} = \frac{1}{4}$

For each problem, find the equation of the tangent line to the function at the given point.

21) $y = -2x^2 - 2$; $(-1, -4)$

$y' = -4x$

$f(-1) = -4$ $\quad$ tan line $y = -4 + 4(x + 1)$

$f'(-1) = 4$ $\quad y = 4x$

22) $y = \dfrac{1}{x + 2}$; $\left(0, \frac{1}{2}\right)$ $\quad y' = \dfrac{0(x+2) - 1(1)}{(x+2)^2} = \dfrac{-1}{(x+2)^2}$

$f(0) = \frac{1}{2}$ $\quad$ tan line $y = \frac{1}{2} + \frac{-1}{4}(x - 0)$

$f'(0) = \frac{-1}{4}$ $\quad y = -\frac{1}{4}x + \frac{1}{2}$

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3) $f(x) = \sqrt{-3x - 3}$

4) $f(x) = -\dfrac{1}{x - 1}$

Differentiate each function with respect to *x*. Problems may contain constants a, b, and c.

5) $f(x) = x^5$

6) $f(x) = \dfrac{4}{x^3}$

7) $f(x) = -5x^{-2}$

8) $f(x) = -\dfrac{4}{5}x^3$

9) $f(x) = 4\sqrt[4]{x}$

10) $f(x) = -4ax^{4b}$

11) $f(x) = \frac{2}{3}x^{3b}$

12) $f(x) = 3b\sqrt[4]{x}$

Differentiate each function with respect to *x*.

13) $y = (2x^5 - 4)(-2x^2 - 1)$

14) $y = (4x^5 - 4)(-x^2 + 2)$

15) $y = (-2\sqrt[3]{x} - 4)(2x^4 - 5x^2 - 1)$

16) $y = \frac{5}{x^3 + 4}$

17) $y = \dfrac{3x^5 - 3x^2 + 2}{4x^2 - 3}$

18) $y = \dfrac{5x^3 - 5}{4 + \dfrac{2}{x^3}}$

For each problem, find the instantaneous rate of change of the function at the given value.

19) $y = -x^2 + x - 1;\ \ 2$

20) $y = -\dfrac{1}{x};\ \ 2$

For each problem, find the equation of the tangent line to the function at the given point.

21) $y = -2x^2 - 2;\ (-1, -4)$

22) $y = \dfrac{1}{x + 2};\ \left(0, \dfrac{1}{2}\right)$