Key

Example 2

Simplify each expression without using your calculator.

1. $64^{1/3}$ 4
2. $(-64)^{1/3}$ -4
3. $-64^{1/3}$ -4
4. $(-25)^{1/2}$ 5i
5. $25^{1/2}$ 5
6. $-25^{1/2}$ -5
7. $25^{-1/2}$ $\frac{1}{5}$
8. $16^{1/4}$ 2
9. $-16^{1/4}$ -2
10. $(-16)^{1/4}$ 2i
11. $16^{-1/4}$ $\frac{1}{2}$
12. $(-32)^{1/5}$ -2
13. $-(-32)^{-1/5}$ $\frac{1}{2}$

Rational Exponents

If m and n are integers, and $a^{1/n}$ is a real number, then

1. $a^{m/n} = (a^m)^{1/n} = (a^{1/n})^m$

2. $a^{-m/n} = \dfrac{1}{a^{m/n}}$

Example 3

Simplify each expression without using your calculator.

1. $16^{3/2}$ 64
2. $16^{-3/2}$ $\frac{1}{64}$
3. $(-64)^{2/3}$ 16
4. $-64^{2/3}$ -16
5. $-64^{-2/3}$ $-\frac{1}{16}$
6. $8^{2/3}$ 4
7. $8^{-2/3}$ $\frac{1}{4}$
8. $-8^{2/3}$ -4
9. $-8^{-2/3}$ $-\frac{1}{4}$
10. $(-8)^{2/3}$ 4
11. $(-8)^{-2/3}$ $\frac{1}{4}$

Example 2

Simplify each expression without using your calculator.

1. $64^{1/3}$

2. $(-64)^{1/3}$

3. $-64^{1/3}$

4. $(-25)^{1/2}$

5. $25^{1/2}$

6. $-25^{1/2}$

7. $25^{-1/2}$

8. $16^{1/4}$

9. $-16^{1/4}$

10. $(-16)^{1/4}$

11. $16^{-1/4}$

12. $(-32)^{1/5}$

13. $-(-32)^{-1/5}$

Rational Exponents

If m and n are integers, and $a^{1/n}$ is a real number, then

1. $a^{m/n} = (a^m)^{1/n}$
   $= (a^{1/n})^m$

2. $a^{-m/n} = \dfrac{1}{a^{m/n}}$

Example 3

Simplify each expression without using your calculator.

1. $16^{3/2}$

2. $16^{-3/2}$

3. $(-64)^{2/3}$

4. $-64^{2/3}$

5. $-64^{-2/3}$

6. $8^{2/3}$

7. $8^{-2/3}$

8. $-8^{2/3}$

9. $-8^{-2/3}$

10. $(-8)^{2/3}$

11. $(-8)^{-2/3}$

Properties of Exponents – A Summary

Let a and b be nonzero numbers, and m, n and p be integers.

1. $b^m b^n = b^{m+n}$
2. $\dfrac{b^m}{b^n} = b^{m-n}$
3. $(b^m)^n = b^{m \cdot n}$
4. $(a^m b^n)^p = a^{mp} b^{np}$
5. $\left(\dfrac{a^m}{b^n}\right)^p = \dfrac{a^{mp}}{b^{np}}$
6. $b^0 = 1$
7. $b^{-n} = \dfrac{1}{b^n}$ and $\dfrac{1}{b^{-n}} = b^n$

Example 4

1. Simplify $(4b^6)^{3/2}$ $8b^9$

2. Simplify $\dfrac{b^{2/7}}{b^{-3/7}}$ $b^{5/7} = \sqrt[7]{b^5}$

3. Simplify $b^{2/3}b^{1/2}$ $b^{4/6}\, b^{3/6} = b^{7/6} = \sqrt[6]{b^7} = b\sqrt[6]{b}$ (add)

4. Simplify $\left(\dfrac{32b^2}{b^{12}}\right)^{2/5}$ $\dfrac{4}{b^4}$

$\left(\dfrac{32}{b^{10}}\right)^{2/5}$

Example 5

a. Simplify $\dfrac{(81b^6c^{20})^{1/2}}{(27b^{12}c^9)^{2/3}}$ $= \dfrac{9b^3c^{10}}{9b^8c^6} = \dfrac{c^4}{b^5}$

b. Simplify $(4xyz)^{1/4}(4xyz)^{3/4}$ $= (4xyz)^{1/4+3/4} = (4xyz)^1$

$4xyz$

c. Simplify $(-8a^3b^6)^{1/3}(4a^8b^4)^{-1/2}$

$\dfrac{(-2ab^2)\,1}{2a^4b^2} = -\dfrac{1}{a^3}$

Properties of Exponents – A Summary

Let a and b be nonzero numbers, and m, n and p be integers.

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5. $\left(\dfrac{a^m}{b^n}\right)^p = \dfrac{a^{mp}}{b^{np}}$
6. $b^0 = 1$
7. $b^{-n} = \dfrac{1}{b^n}$ and $\dfrac{1}{b^{-n}} = b^n$

Example 4

1. Simplify $(4b^6)^{3/2}$

2. Simplify $\dfrac{b^{2/7}}{b^{-3/7}}$

3. Simplify $b^{2/3} b^{1/2}$

4. Simplify $\left(\dfrac{32b^2}{b^{12}}\right)^{2/5}$

Example 5

a. Simplify $\dfrac{(81b^6c^{20})^{1/2}}{(27b^{12}c^9)^{2/3}}$

b. Simplify $(4xyz)^{1/4}(4xyz)^{3/4}$

c. Simplify $(-8a^3b^6)^{1/3}(4a^8b^4)^{-1/2}$