3-Related Rates Practice #2

# Practice Sheet – Related Rates Problems

1. Gas is escaping from a spherical balloon at the rate of 2 $ft^3/min$. How fast is the surface area changing when the radius is 12 *ft*?

2. Sand falling from a chute forms a conical pile whose height is always equal to $\frac{4}{3}$ the radius of the base. How fast is the volume changing when the radius of the base is 3 *ft* and is increasing at the rate of 3 *in/min*?

3. Two parallel sides of a rectangle are being lengthened at the rate of 2 *in/sec*, while the other two sides are shortened in such a way that the figure remains a rectangle with constant area of 50 $in^2$. What is the rate of change of the perimeter when the length of an increasing side is 5 *in*? Is the perimeter increasing or decreasing?

4. A man 5 feet tall walks at the rate 4 *ft/sec* directly away from a street light which is 20 feet above the street. At what rate is the length of his shadow changing? Is the length increasing or decreasing?

5. Water is being withdrawn from a conical reservoir 3 feet in radius and 10 feet deep at a rate of 4 $ft^3/min$. How fast is the surface falling when the depth of the water is 6 feet? How fast is the area of the surface decreasing at this instant?

6. A train, starting at 11am, travels east at 45 *mph* while another, starting at noon from the same point, travels south at 60 *mph*. How fast are they separating at 3pm?

7. A young child is flying a kite horizontally 120 *ft* above the ground. The child lets out 2.5 feet of string per second. If we assume that there is no sag in the string, at what speed is the kite moving when there is 130 *ft* of string out?

8. Oil spilled from a ruptured tanker spreads out in a circle whose area increases at a constant rate of 6 $mi^2/hr$. How fast is the radius of the spill increasing when the area is 9 $mi^2$?

9. Ship *A* is 15 miles east of *P* and is moving west at 20 *mph*; ship *B* is 60 miles south of *P* and is moving north at 15 *mph*. At what rate is the distance between them changing after 1 hour? Is the distance increasing or decreasing?

10. The radius of a right circular cylinder is increasing at a rate of 2 *in/min* and the height is decreasing at a rate of 3 *in/min*. At what rate is the volume changing when the radius is 8 *in* and the height is 12 *in*? Is the volume increasing or decreasing?

# 3- Related Rates Practice #2 Key

## Practice Sheet

① 

$\frac{dV}{dt} = -2\ \frac{ft^3}{min}$ $\quad \frac{d(SA)}{dt} = ?$ $\quad r=12$

$V = \frac{4}{3}\pi r^3$ $\qquad SA = 4\pi r^2$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ $\qquad \frac{d(SA)}{dt} = 8\pi r \frac{dr}{dt}$

$-2 = 4\pi(12)^2 \frac{dr}{dt}$ $\qquad \frac{d(SA)}{dt} = 8\pi(12)\left(\frac{-1}{288\pi}\right)$

$\frac{dr}{dt} = \frac{-1}{288\pi}$ $\qquad \frac{d(SA)}{dt} = -\frac{1}{3}$

decreasing at $1\ ft/3\ min$

②


$\frac{dV}{dt} = ?$ $\quad r=3$ $\quad \frac{dr}{dt} = 3\ in/min = .25\ ft/min$

$h = \frac{4}{3}r$

$\frac{dh}{dt} = \frac{4}{3}(.25) = \frac{1}{3}$

$V = \frac{1}{3}\pi r^2 h$

$\frac{dV}{dt} = \frac{2}{3}\pi r \frac{dr}{dt} h + \frac{1}{3}\pi r^2 \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{2}{3}\pi(3)(.25)\left(\frac{4}{3}(3)\right) + \frac{1}{3}\pi(3)^2\left(\frac{1}{3}\right)$

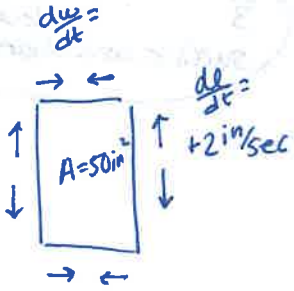
$\frac{dV}{dt} = 2\pi + 1\pi = 3\pi\ ft^3/min$ volume increasing

$V = \frac{1}{3}\pi r^2\left(\frac{4}{3}r\right)$

$V = \frac{4}{9}\pi r^3$

$\frac{dV}{dt} = \frac{4}{3}\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = \frac{4}{3}\pi(3)^2(3)$

③ 


$\frac{dp}{dt} = ?$ $\quad l=5$

$A = lw$

$p = 2l + 2w$

$A = lw$

$50 = lw$

$w = \frac{50}{l} = 50l^{-1}$

$\frac{dw}{dt} = -50l^{-2}\frac{dl}{dt}$

$\frac{dp}{dt} = 2\frac{dl}{dt} + 2\frac{dw}{dt}$

$\frac{dp}{dt} = 2(2) + 2(-50)(5)^{-2}(2)$

$\frac{dp}{dt} = 4 - 8 = -4$

perimeter is decreasing at 4in/sec.

$l' = 2$
$A = 50$
$w = 10$
$p' = ?$
$l = 5$

$A = lw$

$A' = l'w + lw'$

$0 = (2)(10) + (5)w'$

$-20 = 5w'$

$-4 = w'$

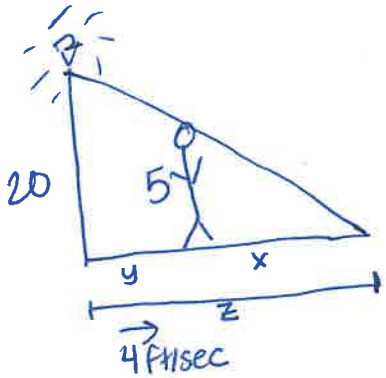
$p = 2l + 2w$

$p' = 2l' + 2w'$

$p' = 2(2) + 2(-4)$

$p' = -4$

(4)




$\frac{5}{x} = \frac{20}{x+y}$

$5x + 5y = 20x$

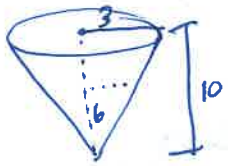
$5y = 15x$

$y = 3x$

$\frac{dy}{dt} = 3\frac{dx}{dt}$

$4 = 3\frac{dx}{dt}$ $\quad \frac{dx}{dt} = \frac{4}{3}$ ft/sec increasing length of shadow

(5)




$\frac{dV}{dt} = 4$ ft$^3$/min $\quad \frac{d(SA)}{dt} = ?$

$V = \frac{1}{3}\pi r^2 h$

$\frac{dV}{dt} = \frac{2}{3}\pi r \frac{dr}{dt} + \frac{1}{3}\pi r^2 \frac{dh}{dt}$ don't know $\frac{dr}{dt}$ or r ... replace with $r = \frac{3h}{10}$

$\frac{3}{10} = \frac{r}{h}$

$3h = 10r$

$r = \frac{3h}{10}$

$V = \frac{1}{3}\pi\left(\frac{3h}{10}\right)^2 h$

$V = \frac{1}{3}\pi\frac{9h^2}{100}\cdot h$

$V = \frac{3\pi}{100}h^3$

$\frac{dV}{dt} = \frac{9\pi}{100}h^2\frac{dh}{dt}$

$-4 = \frac{9\pi}{100}(6)^2\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{-100}{81\pi}$ ft/min decreasing volume

surface is a circle

$A = \pi r^2$

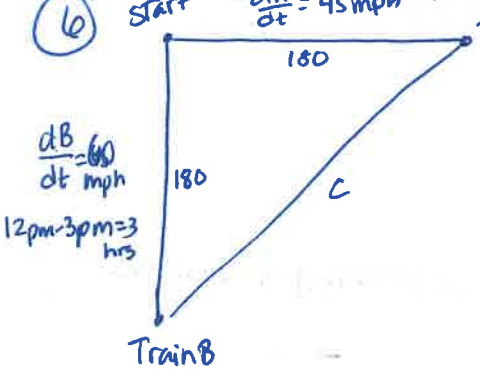
$A = \pi\left(\frac{3h}{10}\right)^2$

$A = \pi\cdot\frac{9h^2}{100}$

$\frac{dA}{dt} = \frac{18\pi}{100}h\frac{dh}{dt}$

$\frac{dA}{dt} = \frac{18\pi}{100}\cdot 6\cdot\frac{-100}{81\pi} = \frac{-4}{3}$ ft$^2$/min decreasing surface area changing

(6)




$\frac{dA}{dt} = 45$ mph (11am-3pm = 4 hrs)

$\frac{dB}{dt} = 60$ mph

12pm-3pm = 3 hrs

$a^2 + b^2 = c^2$

$180^2 + 180^2 = c^2$

$32400 + 32400 = c^2$

$64800 = c^2$

$254.558 \approx 180\sqrt{2} = c$

$a^2 + b^2 = c^2$

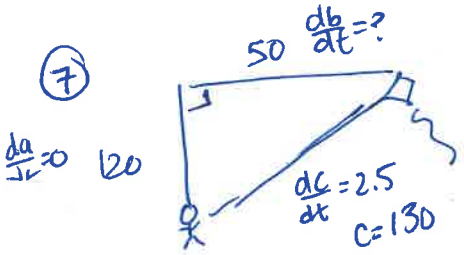
$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$

$2(180)(45) + 2(180)(60) = 2(180\sqrt{2})\frac{dc}{dt}$

$37800 = 360\sqrt{2}\frac{dc}{dt}$

$\frac{dc}{dt} = \frac{105\sqrt{2}}{2} \approx 74.25$ mph

(7)




$a^2+b^2=c^2$

$120^2+b^2=130^2$

$14400+b^2=16900$

$b^2=2500$

$b=50$

$2a\frac{da}{dt}+2b\frac{db}{dt}=2c\frac{dc}{dt}$

$2(120)(0)+2(50)\frac{db}{dt}=2(130)(2.5)$

$100\frac{db}{dt}=650$

$\frac{db}{dt}=6.5$ ft/sec

(8)

$\frac{dA}{dt}=6\ mi^2/hr$ $\frac{dr}{dt}=?$

$A=9mi^2$

$A=\pi r^2$

$\frac{dA}{dt}=2\pi r\frac{dr}{dt}$

$6=2\pi\left(\frac{\sqrt{9}}{\sqrt{\pi}}\right)\frac{dr}{dt}$

$6=6\sqrt{\pi}\frac{dr}{dt}$

$\frac{1}{\sqrt{\pi}}=\frac{dr}{dt}$

radius is increasing at $\frac{1}{\sqrt{\pi}}$

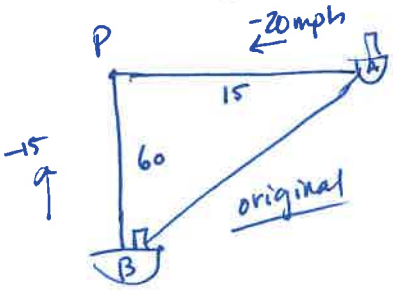
$A=\pi r^2$

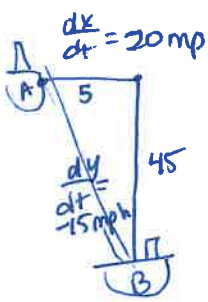
$\frac{A}{\pi}=r^2$

$r=\frac{\sqrt{A}}{\sqrt{\pi}}$

$\frac{dA}{dt}=2\pi\left(\frac{\sqrt{A}}{\sqrt{\pi}}\right)\frac{dr}{dt}$

(9)




$5^2+45^2=z^2$

$25+2025=z^2$

$2050=z^2$

$z=5\sqrt{82}$

$2x\frac{dx}{dt}+2y\frac{dy}{dt}=2z\frac{dz}{dt}$

$2(5)(20)+2(45)(-15)=2(5\sqrt{82})\frac{dz}{dt}$

$200-1350=10\sqrt{82}\frac{dz}{dt}$

$-1150=10\sqrt{82}\frac{dz}{dt}$

$\frac{dz}{dt}=-12.7$ mph decreasing dist.

(10)

$\frac{dr}{dt}=2\ in/min$ $r=8$

$\frac{dh}{dt}=-3\ in/min$ $h=12$

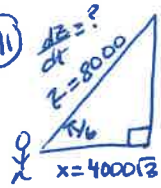
$V=\pi r^2 h$

$\frac{dV}{dt}=2\pi r\frac{dr}{dt}+\pi r^2\frac{dh}{dt}$

$\frac{dV}{dt}=2\pi(8)(2)(12)+\pi(8)^2(-3)$

$\frac{dV}{dt}=192\pi\ in^3/min$ increasing

(11)




$x=4000\sqrt{3}\approx 6928.203$

$\frac{dx}{dt}=440$ ft/sec

$\tan(\pi/6)=\frac{4000}{x}$

$x=4000\sqrt{3}$

$\tan\theta=yx^{-1}$

$\sec^2\theta\frac{d\theta}{dt}=\frac{dy}{dt}x^{-1}+y(-1x^{-2})\frac{dx}{dt}$

$\sec^2(\pi/6)\frac{d\theta}{dt}=0+(4000)(-1\cdot 6928.203^{-2})(440)$

$\frac{d\theta}{dt}=-0.0275$ rad/sec

$4000^2+(4000\sqrt{3})^2=z^2$

$8000=z$

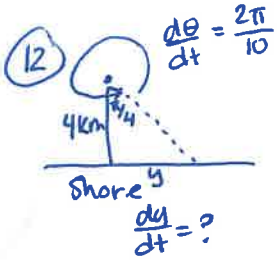
$2x\frac{dx}{dt}+2y\frac{dy}{dt}=2z\frac{dz}{dt}$

$2(4000\sqrt{3})(440)+2(4000)(0)=2(8000)\frac{dz}{dt}$

$3520000\sqrt{3}=16000\frac{dz}{dt}$

$\frac{dz}{dt}=381.05$ ft/hr

(12)




$\tan\theta=\frac{y}{4}$

$\sec^2\theta\frac{d\theta}{dt}=\frac{1}{4}\frac{dy}{dt}$

$\sec^2(\pi/4)(\pi/5)=\frac{1}{4}\frac{dy}{dt}$

$\frac{dy}{dt}=\frac{5\pi}{5}$ km/sec