

Extra Practice - Chain Rule

Differentiate each function with respect to x .

1) $f(x) = (-x^2 - 5)^{-4}$

$$f'(x) = -4(-x^2 - 5)^{-5} \cdot -2x$$

$$= \frac{8x}{(-x^2 - 5)^5}$$

$$\text{or } f'(x) = 8x(-x^2 - 5)^5$$

2) $f(x) = (4x^2 - 3)^{-4}$

$$f'(x) = -4(4x^2 - 3)^{-5} \cdot 8x$$

$$= -\frac{32x}{(4x^2 - 3)^5}$$

$$f'(x) = -32x(4x^2 - 3)^5$$

3) $f(x) = (3x^5 + 4)^{-2}$

$$f'(x) = -2(3x^5 + 4)^{-3} \cdot 15x^4$$

$$= -\frac{30x^4}{(3x^5 + 4)^3}$$

$$f'(x) = -30x^4(3x^5 + 4)^3$$

4) $f(x) = (-5x^5 - 3)^3$

$$f'(x) = 3(-5x^5 - 3)^2 \cdot -25x^4$$

$$= -75x^4(-5x^5 - 3)^2$$

5) $f(x) = \sqrt{-x^5 - 3}$

$$f'(x) = \frac{1}{2}(-x^5 - 3)^{-\frac{1}{2}} \cdot -5x^4$$

$$= -\frac{5x^4}{2(-x^5 - 3)^{\frac{1}{2}}}$$

$$f'(x) = -\frac{5}{2}x^4(-x^5 - 3)^{\frac{1}{2}}$$

6) $f(x) = (x^2 - 5)^{\frac{1}{5}}$

$$f'(x) = \frac{1}{5}(x^2 - 5)^{-\frac{4}{5}} \cdot 2x$$

$$= \frac{2x}{5(x^2 - 5)^{\frac{4}{5}}}$$

$$f'(x) = \frac{2x}{5}(x^2 - 5)^{-4/5}$$

$$7) f(x) = \sqrt[3]{5x^4 - 3}$$

$$f'(x) = \frac{1}{3}(5x^4 - 3)^{-\frac{2}{3}} \cdot 20x^3$$

$$= \frac{20x^3}{3(5x^4 - 3)^{\frac{2}{3}}}$$

$$f'(x) = \frac{20x^3}{3}(5x^4 - 3)^{-\frac{2}{3}}$$

$$f'(x) = 2(-2x+1) \cdot (-2) = -4(-2x+1)$$

$$g'(x) = (-5x^4)$$

$$8) f(x) = \boxed{(-2x+1)^2} \cdot \boxed{-x^5+4}$$

$$f'(x) = (-2x+1)^2 \cdot -5x^4 + (-x^5+4) \cdot 2(-2x+1) \cdot -2$$

$$= (-2x+1)(14x^5 - 5x^4 - 16)$$

$$f'(x) = -4(-2x+1)(-x^5+4) - 5x^4(-2x+1)^2$$

$$9) f(x) = \sqrt[4]{\frac{5x^2+4}{-5x+3}}$$

$$\frac{(5x^2+4)^{1/4}}{(-5x+3)^{1/4}}$$

$$\frac{1}{4}(5x^2+4)^{-3/4} \cdot 10x$$

$$\frac{1}{4}(-5x+3)^{-5/4} \cdot (-5)$$

$$f'(x) = \frac{1}{4} \cdot \left(\frac{5x^2+4}{-5x+3}\right)^{-\frac{3}{4}} \cdot \frac{(-5x+3) \cdot 10x - (5x^2+4) \cdot -5}{(-5x+3)^2}$$

$$= \frac{5(-5x^2+6x+4)}{4(5x^2+4)^{\frac{3}{4}} \cdot (-5x+3)^{\frac{5}{4}}}$$

$$f'(x) = \frac{\frac{5}{2}x(5x^2+4)^{-3/4}(-5x+3)^{1/4} + \frac{5}{4}(5x^2+4)^{1/4}(-5x+3)^{-5/4}}{(-5x+3)^{5/2}}$$

or sheet

$$10) f(x) = \frac{-5x^3+2}{(4x^5-3)^{\frac{1}{3}}}$$

$$-15x^2$$

$$\frac{1}{3}(4x^5-3)^{-2/3} (20x^4)$$

$$f'(x) = \frac{(4x^5-3)^{\frac{1}{3}} \cdot -15x^2 - (-5x^3+2) \cdot \frac{1}{3}(4x^5-3)^{-\frac{2}{3}} \cdot 20x^4}{\left((4x^5-3)^{\frac{1}{3}}\right)^2}$$

$$= \frac{5x^2(-16x^5+27-8x^2)}{3(4x^5-3)^{\frac{4}{3}}}$$

$$f'(x) = \text{see sheet}$$

$$11) f(x) = (\sqrt[4]{x+5} - 5)^{-5}$$

$$\begin{aligned} f'(x) &= -5 \left((x+5)^{\frac{1}{4}} - 5 \right)^{-6} \cdot \frac{1}{4} (x+5)^{-\frac{3}{4}} \\ &= \frac{5}{4 \left((x+5)^{\frac{1}{4}} - 5 \right)^6 \cdot (x+5)^{\frac{3}{4}}} \end{aligned}$$

$$12) f(x) = \sqrt{(-x^3+2)^4+1}$$

$$\begin{aligned} f'(x) &= \frac{1}{2} \left((-x^3+2)^4+1 \right)^{-\frac{1}{2}} \cdot 4(-x^3+2)^3 \cdot -3x^2 \\ &= \frac{6x^2(-x^3+2)^3}{\left((-x^3+2)^4+1 \right)^{\frac{1}{2}}} \end{aligned}$$

$$13) f(x) = \left((2x^5+5)^{-5} - 1 \right)^{-2}$$

$$\begin{aligned} f'(x) &= -2 \left((2x^5+5)^{-5} - 1 \right)^{-3} \cdot -5(2x^5+5)^{-6} \cdot 10x^4 \\ &= \frac{100x^4(2x^5+5)^9}{\left(-(2x^5+5)^5 + 1 \right)^3} \end{aligned}$$

$$14) f(x) = \left(\sqrt{-x^5+5} - 2 \right)^{\frac{1}{5}}$$

$$\begin{aligned} f'(x) &= \frac{1}{5} \left((-x^5+5)^{\frac{1}{2}} - 2 \right)^{-\frac{4}{5}} \cdot \frac{1}{2} (-x^5+5)^{-\frac{1}{2}} \cdot -5x^4 \\ &= \frac{x^4}{2 \left((-x^5+5)^{\frac{1}{2}} - 2 \right)^{\frac{4}{5}} \cdot (-x^5+5)^{\frac{1}{2}}} \end{aligned}$$

$$15) f(x) = \frac{(x^4+2)^{-2}}{(-5x+2)^4}$$

$$\begin{aligned} f'(x) &= \frac{(-5x+2)^4 \cdot -2(x^4+2)^{-3} \cdot 4x^3 - (x^4+2)^{-2} \cdot 4(-5x+2)^3 \cdot -5}{\left((-5x+2)^4 \right)^2} \\ &= \frac{4(15x^4 - 4x^3 + 10)}{(x^4+2)^3 \cdot (-5x+2)^5} \end{aligned}$$

$$16) f(x) = (-2x^2-5)^{\frac{1}{5}} \cdot (5x^3-4)^{-4}$$

$$\begin{aligned} f'(x) &= (-2x^2-5)^{\frac{1}{5}} \cdot -4(5x^3-4)^{-5} \cdot 15x^2 + (5x^3-4)^{-4} \cdot \frac{1}{5}(-2x^2-5)^{-\frac{4}{5}} \cdot -4x \\ &= \frac{4x(145x^3+375x+4)}{5(-2x^2-5)^{\frac{4}{5}} \cdot (5x^3-4)^5} \end{aligned}$$

$$17) f(x) = \frac{(x-4)^{\frac{1}{2}}}{(-x^5+2)^4}$$

$$\begin{aligned} f'(x) &= \frac{(-x^5+2)^4 \cdot \frac{1}{2}(x-4)^{-\frac{1}{2}} - (x-4)^{\frac{1}{2}} \cdot 4(-x^5+2)^3 \cdot -5x^4}{((-x^5+2)^4)^2} \\ &= \frac{39x^5 - 160x^4 + 2}{2(x-4)^{\frac{1}{2}} \cdot (-x^5+2)^5} \end{aligned}$$

$$18) f(x) = \frac{(-x^3-5)^{-5}}{(5x^5-4)^2}$$

$$\begin{aligned} f'(x) &= \frac{(5x^5-4)^2 \cdot -5(-x^3-5)^{-6} \cdot -3x^2 - (-x^3-5)^{-5} \cdot 2(5x^5-4) \cdot 25x^4}{((5x^5-4)^2)^2} \\ &= \frac{5x^2(25x^5 - 12 + 50x^2)}{(-x^3-5)^6 \cdot (5x^5-4)^3} \end{aligned}$$

$$9. \quad \frac{1}{4} \left(\frac{5x^2+4}{-5x+3} \right)^{-3/4} \left(\frac{10x(-5x+3) - (-5)(5x^2+4)}{(-5x+3)^2} \right)$$

$$\left(\frac{-50x^2+30x+25x^2+20}{(-5x+3)^2} \right)$$

$$f'(x) = \frac{1}{4} \left(\frac{5x^2+4}{-5x+3} \right)^{-3/4} \left(\frac{-25x^2+30x+20}{(-5x+3)^2} \right)$$

$$10. \quad \frac{-15x^2(4x^5-3)^{1/3} - (-5x^3+2) \left(\frac{1}{3} (4x^5-3)^{-2/3} (20x^4) \right)}{(4x^5-3)^{4/3}}$$

$$f'(x) = \frac{-15x^2(4x^5-3)^{1/3} - \frac{20}{3}x^4(-5x^3+2)(4x^5-3)^{-2/3}}{(4x^5-3)^{4/3}}$$

$$11. \quad -5 \left(\sqrt[4]{x+5} - 5 \right)^{-6} \left(\frac{1}{4} (x+5)^{-3/4} \right)$$

$$f'(x) = -\frac{5}{4} \left(\sqrt[4]{x+5} - 5 \right)^{-6} (x+5)^{-3/4}$$

$$12. \quad \frac{1}{2} \left[(-x^3+2)^4 + 1 \right]^{-1/2} \cdot 4(-x^3+2)^3 \cdot (-3x)$$

$$-6x \left[(-x^3+2)^4 + 1 \right]^{-1/2} (-x^3+2)^3$$

$$13. \quad -2 \left[(2x^5+5)^{-5} - 1 \right]^{-3} \cdot -5(2x^5+5)^{-6} \cdot (10x^4)$$

$$100x^4 \left[(2x^5+5)^{-5} - 1 \right]^{-3} (2x^5+5)^{-6}$$

$$14. \quad \frac{1}{5} \left(\sqrt{-x^5+5} - 2 \right)^{-4/5} \cdot \frac{1}{2} (-x^5+5)^{-1/2} \cdot (-5x^4)$$

$$-\frac{1}{2}x^4 \left(\sqrt{-x^5+5} - 2 \right)^{-4/5} (-x^5+5)^{-1/2}$$

$$(15.) \quad f(x) = -2(x^4+2)^{-3} (4x^3) = -8x^3(x^4+2)^{-3}$$

$$g(x) = 4(-5x+2)^3(-5x) = -20x(-5x+2)^3$$

$$Q \quad \frac{-8x^3(x^4+2)^{-3}(-5x+2)^4 + 20x(x^4+2)^{-2}(-5x+2)^3}{(-5x+2)^8}$$

$$(16.) \quad \frac{\frac{1}{5}(-2x^2-5)^{-4/5} \cdot (-4x)(5x^3-4)^{-4} + (-2x^2-5)^{1/5} \cdot -4(5x^3-4)^{-5}(15x^2) - \frac{4}{5}x(-2x^2-5)^{-4/5}(5x^3-4)^{-4} - 60x^2(-2x^2-5)^{1/5}(5x^3-4)^{-5}}{(-2x^2-5)^8}$$

$$(17.) \quad \frac{\frac{1}{2}(x-4)^{-1/2}(-x^5+2)^4 - (x-4)^{1/2} \cdot 4(-x^5+2)^3 \cdot -5x^4}{(-x^5+2)^8}$$

$$\frac{\frac{1}{2}(x-4)^{-1/2}(-x^5+2)^4 + 20x^4(x-4)^{1/2}(-x^5+2)^3}{(-x^5+2)^8}$$

$$(18.) \quad \frac{-5(-x^3-5)^{-6}(-3x^2)(5x^5-4)^2 - (-x^3-5)^{-5} \cdot 2(5x^5-4)(25x^4)}{(5x^5-4)^4}$$

$$\frac{15x^2(-x^3-5)^{-6}(5x^5-4)^2 - 50x^4(-x^3-5)^{-5}(5x^5-4)}{(5x^5-4)^4}$$

Factoring: Sum/Difference of Cubes

t) $x^3 - 8$

u) $27b^3 + 1$

v) $3x^6 + 192$

1) $-48a + 6a^2$

2) $-35p^2 - 21$

3) $-30x - 10x^2 - 20x^7$

4) $-30n^4 - 48n^3 + 30n^2 + 60n$

5) $25m^3 - 35m^2 - 20m + 28$

6) $12n^3 + 3n^2 + 28n + 7$