Key

Calculus

Finding Limits Graphically and Numerically

Obj: -Estimate a limit using a numerical or graphical approach
- Learn different ways that a limit can fail to exist

1. Find $\lim_{x \to 2} \frac{x^2 - 3x + 2}{x - 2}$ by

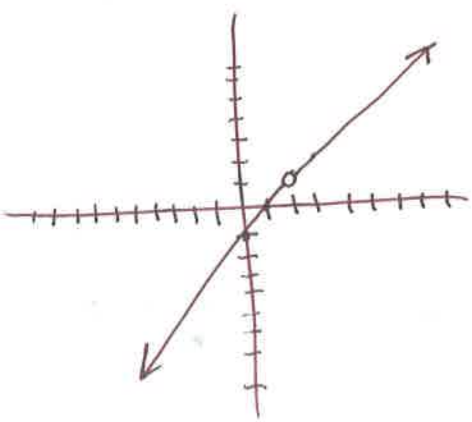
a) constructing a table

x	1.75	1.9	1.99	1.999	2	2.001	2.01	2.1	2.25
$f(x)$	.75	.9	.99	.999	DNE	1.001	1.01	1.1	1.25

b) drawing a graph

$\frac{x^2-3x+2}{x-2} = \frac{(x-2)(x-1)}{(x-2)} = x-1$ but $x \neq 2$

-before graphing, try to simplify function
-note that the simplified version is not the exact equivalence. Why not?



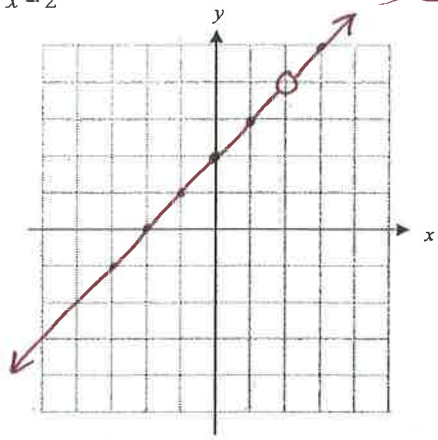
FINDING LIMITS GRAPHICALLY AND NUMERICALLY

An Introduction to Limits

Example: Sketch the graph of $f(x) = \frac{x^2 - 4}{x - 2}$; $x \neq 2$

$\frac{(x-2)(x+2)}{(x-2)} = x+2$

$x \neq 2$



(a) What happens at $x = 2$? DNE

(b) Even though this function is not defined at $x = 2$, we can still examine its behavior close to $x = 2$. Complete the table of values of the function close to $x = 2$.

x approaches 2 from the **left** ⟶ | ⟵ x approaches 2 from the **right**

x	1.5	1.75	1.9	1.99	1.999	2	2.001	2.01	2.1	2.25	2.5
$f(x)$	3.5	3.75	3.9	3.99	3.999	DNE	4.001	4.01	4.1	4.25	4.5

Informal Definition of a Limit

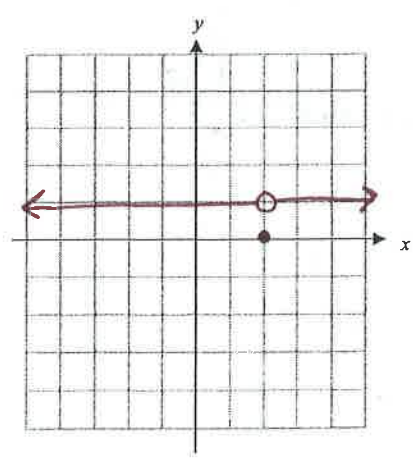
Suppose a function f is defined on an interval around c, but possibly not at the point $x = c$ itself. Suppose that as x becomes sufficiently close to c, $f(x)$ becomes as close to a single number L as we please.
We then say that the **limit of $f(x)$ as x approaches c is L**, and we write

$$\lim_{x \to c} f(x) = L$$

(c) Apply this definition to the function from above to find the $\lim_{x \to 2} f(x)$. $= 4$

Example: Find $\lim_{x \to 2} g(x)$, where g is defined as

$$g(x) = \begin{cases} 1, & x \neq 2 \\ 0, & x = 2 \end{cases}$$



$\lim_{x \to 2} g(x) = 1$

notice that we don't care about $g(2)$!

Finding Limits Graphically and Numerically Calculus Key

There are 3 basic ways to evaluate a limit. So far we have used two of them. In section 3, we will use the third.

1. Numerical approach … Make a table
2. Graphical approach … Draw a graph by hand or using the calculator
3. Analytical approach … Use algebra or calculus (section 3)

The Formal Definition of a Limit

When we say "$f(x)$ becomes as close to L as we please" in the informal definition, we mean that we can specify a maximum distance between $f(x)$ and L. This distance is given by

$$|f(x)-L| = \text{Distance between } f(x) \text{ and } L.$$

We use the Greek letter ε (epsilon) to stand for the maximum distance, so we require

$$|f(x)-L| < \varepsilon.$$

Similarly, we interpret "x becomes sufficiently close to c" to mean

$$|x-c| < \delta,$$

where the Greek letter δ (delta) tells us how close x must be to c. Then

$$\lim_{x\to c} f(x) = L$$

means that we can make the distance $|f(x)-L|$ between the function values and L as small as we like (less than any number $\varepsilon > 0$) by making the distance $|x-c|$ between x and c sufficiently small (less than some $\delta > 0$).

Definition: Suppose a function f is defined on an interval around c, but possibly not at the point $x = c$ itself. Suppose that for any $\varepsilon > 0$ (as small as we like), there is a $\delta > 0$ (sufficiently small) so that

$$\text{if } 0 < |x-c| < \delta, \text{ then } |f(x)-L| < \varepsilon.$$

We then say the limit of $f(x)$ as x approaches c exists and write

$$\lim_{x\to c} f(x) = L.$$

When Limits Do Not Exist

If there does not exist a number L satisfying the condition in the definition, then we say the $\lim_{x\to c} f(x)$ does not exist.

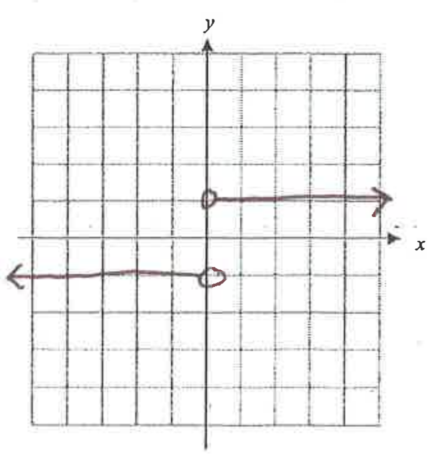
Limits typically fail for three reasons:

1. $f(x)$ approaches a different number from the right side of c than it approaches from the left side.
2. $f(x)$ increases or decreases without bound as x approaches c.
3. $f(x)$ oscillates between two fixed values as x approaches c.

use t-chart or piecewise to graph

Example: Investigate the existence of the following limits.

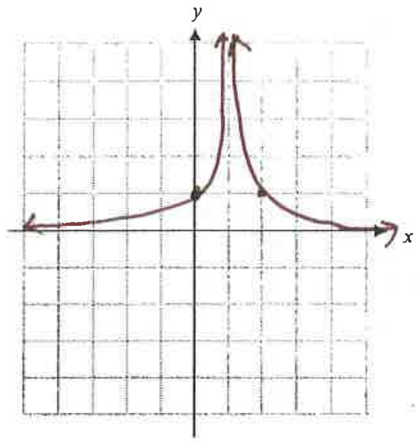
(a) $\lim_{x\to 0} \frac{|x|}{x}$ = DNE



x	y
-3	-1
-2	-1
-1	-1
0	DNE
1	1
2	1
3	1

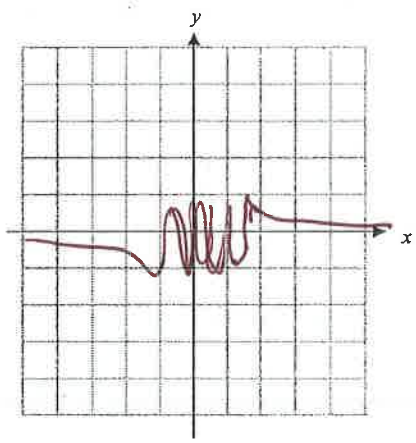
x	−0.5	−0.25	−0.1	−.01	−.001	0	.001	.01	.1	.25	.5
$f(x)$	−1	−1	−1	−1	−1	DNE	1	1	1	1	1

(b) $\lim_{x\to 1} \frac{1}{(x-1)^2}$



x	0	.5	.9	.99	.999	1	1.001	1.01	1.1	1.5	2
$f(x)$	1	4	100	10,000	1,000,000	DNE	1,000,000	10,000	100	4	1

(c) $\lim_{x\to 0} \sin\left(\frac{1}{x}\right)$



x	$\frac{2}{\pi}$	$\frac{2}{3\pi}$	$\frac{2}{5\pi}$	$\frac{2}{7\pi}$	$\frac{2}{9\pi}$	$\frac{2}{11\pi}$	$\frac{2}{13\pi}$	As $x \to 0$
$f(x)$	1	−1	1	−1	1	−1	1	DNE

♫ **(NOTE):** Throughout the text, when you see

$$\lim_{x\to c} f(x) = L,$$

two statements are *implied* → (1) the limit exists, and (2) the limit is L.

Calculus

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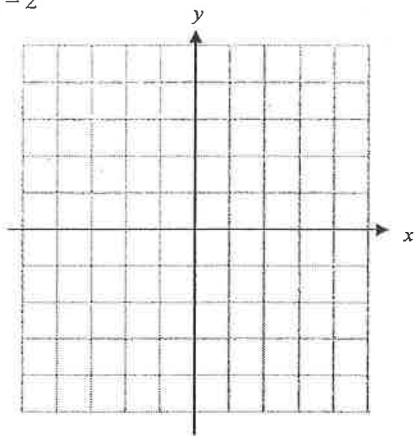
FINDING LIMITS GRAPHICALLY AND NUMERICALLY

An Introduction to Limits

Example: Sketch the graph of $f(x)=\dfrac{x^2-4}{x-2}$ $;x \neq 2$



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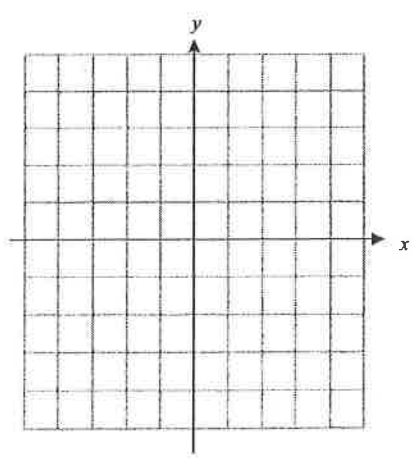

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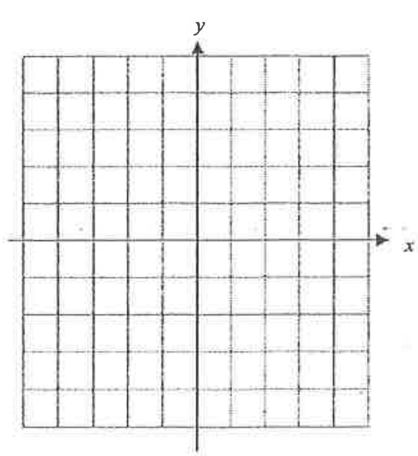
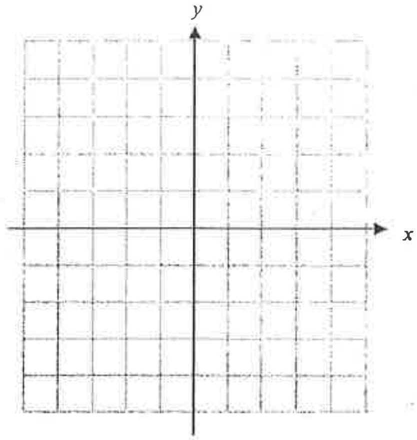
(a) $\lim_{x\to 0}\frac{|x|}{x}$



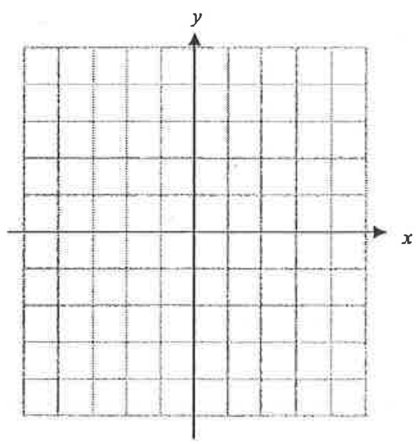
x	−0.5	−0.25	−0.1	−.01	−.001	0	.001	.01	.1	.25	.5
$f(x)$											

(b) $\lim_{x \to 1} \frac{1}{(x-1)^2}$



x	0	.5	.9	.99	.999	1	1.001	1.01	1.1	1.5	2
$f(x)$											

(c) $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$



x	$\frac{2}{\pi}$	$\frac{2}{3\pi}$	$\frac{2}{5\pi}$	$\frac{2}{7\pi}$	$\frac{2}{9\pi}$	$\frac{2}{11\pi}$	$\frac{2}{13\pi}$	As $x \to 0$
$f(x)$								

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Finding Limits Graphically and Numerically

Name: key

Complete the table and use the result to determine the limit.

1. $\lim_{x \to 4} \frac{x-4}{x^2-3x-4}$ = .2

x	3.9	3.99	3.999	4.001	4.01	4.1
f (x)	.20408	.20040	.20004	.19996	.19960	.196078

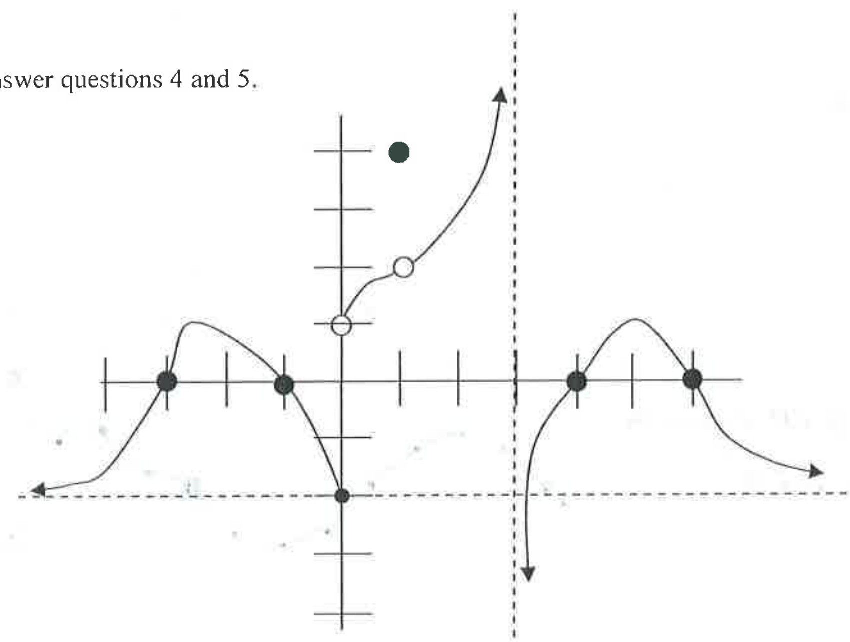
2. $\lim_{x \to 0} \frac{\sin x}{x}$ = 1

x	-.1	-.01	-.001	.001	.01	.1
f (x)	.998334	.999983	.999999	.999999	.999983	.998334

3. $\lim_{x \to 0} \frac{\ln(x+1)}{x}$

x	-.1	-.01	-.001	.001	.01	.1
f (x)	1.05361	1.0050	1.0005	.999500	.995033	.953102

Use the given graph to answer questions 4 and 5.



4. A) $f(0)$ = -2 B) $\lim_{x \to 0} f(x)$ = DNE C) $f(1) = 4$ D) $\lim_{x \to 1} f(x) = 2$ E) What are the zeros for $f(x)$

{-3, -1, 4, 6}

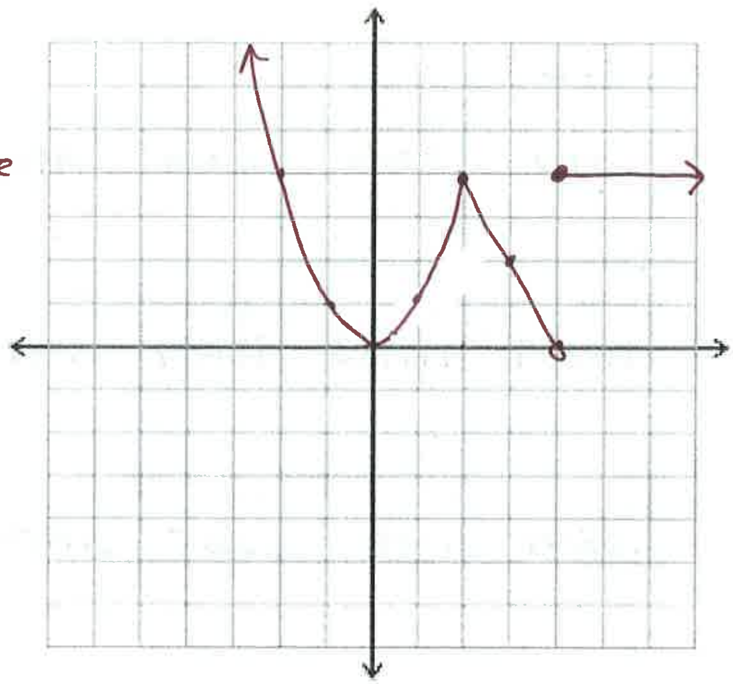
5. A) $f(3)$ = DNE B) $\lim_{x \to 3} f(x)$ DNE C) $f(4) = 0$ D) $\lim_{x \to 4} f(x) = 0$

E) Does $f(x)$ have an inverse **function**? Explain.

No, doesn't pass the horizontal line test

Sketch the graph of f (x). Then identify the values of c for which $\lim_{x \to c} f(x)$ exists.

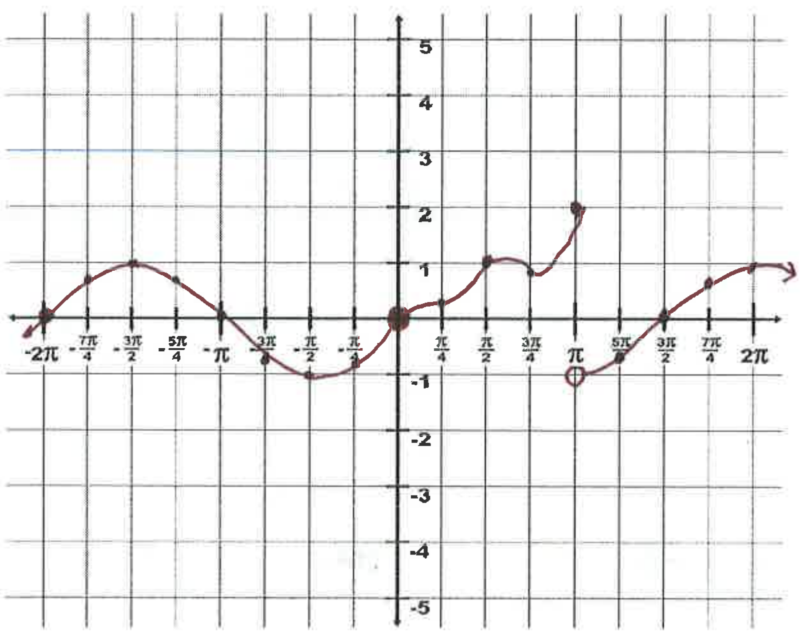
6. $f(x) = \begin{cases} x^2 & x \le 2 \\ 8 - 2x & 2 < x < 4 \\ 4 & x \ge 4 \end{cases}$



$\lim_{x \to c} f(x)$ exists everywhere except $x = 4$

use unit circle

7. $f(x) = \begin{cases} \sin x & x < 0 \\ 1 - \cos x & 0 \le x \le \pi \\ \cos x & x > \pi \end{cases}$



$\lim_{x \to c} f(x)$ exists everywhere except $x = \pi$

Finding Limits Graphically and Numerically Name: ____________________

Complete the table and use the result to determine the limit.

1. $\lim_{x \to 4} \frac{x-4}{x^2-3x-4}$

x	3.9	3.99	3.999	4.001	4.01	4.1
f (x)						

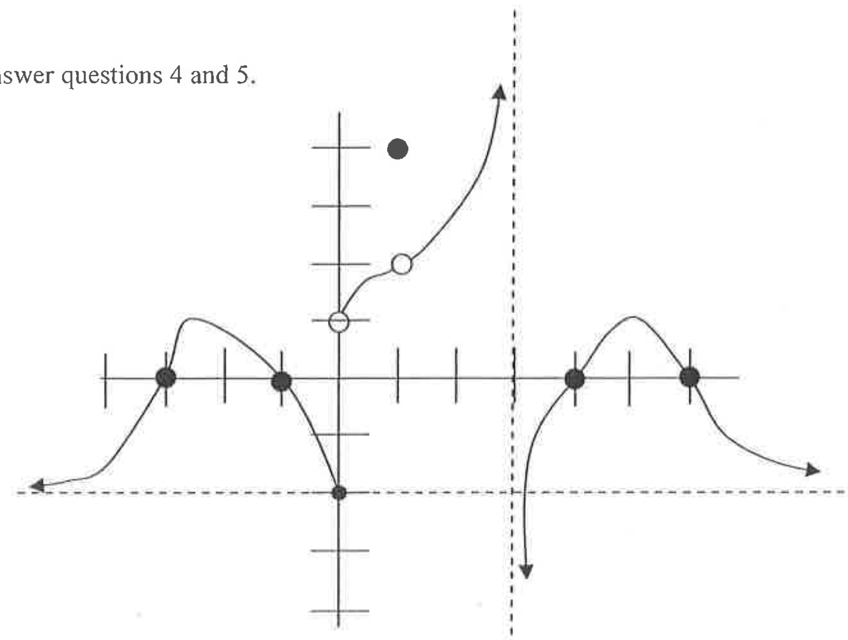
2. $\lim_{x \to 0} \frac{\sin x}{x}$

x	-.1	-.01	-.001	.001	.01	.1
f (x)						

3. $\lim_{x \to 0} \frac{\ln(x+1)}{x}$

x	-.1	-.01	-.001	.001	.01	.1
f (x)						

Use the given graph to answer questions 4 and 5.



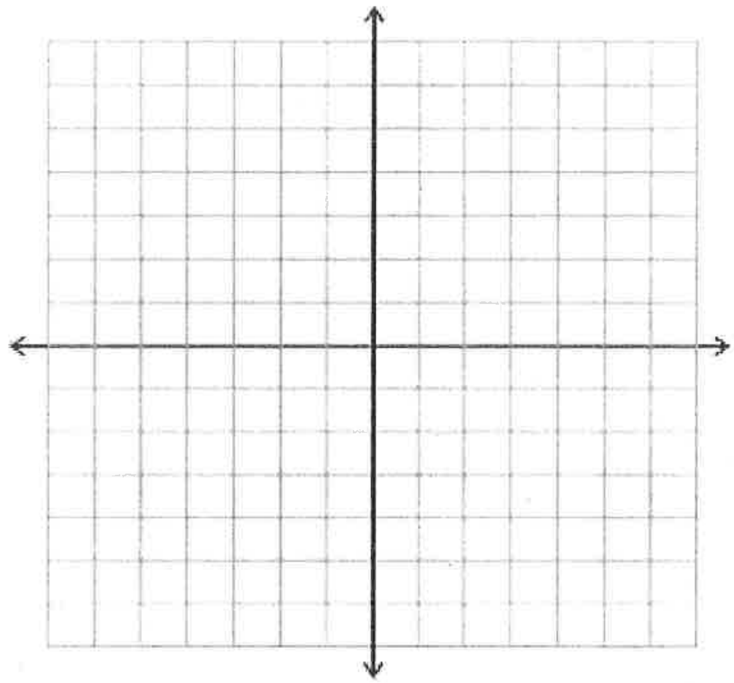
4. A) $f(0) =$ B) $\lim_{x \to 0} f(x) =$ C) $f(1) =$ D) $\lim_{x \to 1} f(x) =$ E) What are the zeros for $f(x)$

5. A) $f(3) =$ B) $\lim_{x \to 3} f(x)$ C) $f(4) =$ D) $\lim_{x \to 4} f(x) =$

E) Does $f(x)$ have an inverse **function**? Explain.

Sketch the graph of f (x). Then identify the values of c for which $\lim_{x \to c} f(x)$ exists.

6. $f(x) = \begin{cases} x^2 & x \le 2 \\ 8 - 2x & 2 < x < 4 \\ 4 & x \ge 4 \end{cases}$



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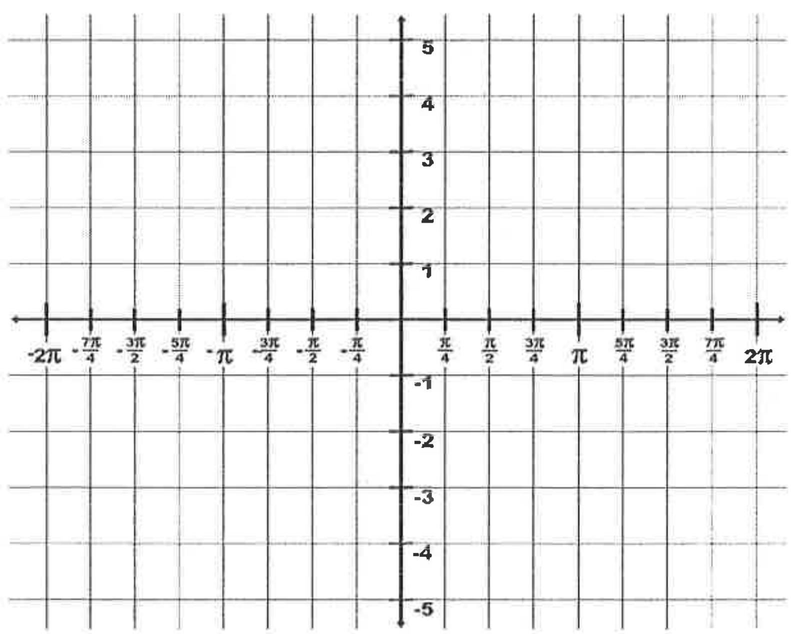

Key

Finding a Limit:

<table>
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Right hand limit : $\lim_{x \to c^+} f(x)$

- the limit of $f(x)$ as x approaches c from the right

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<tr><td>

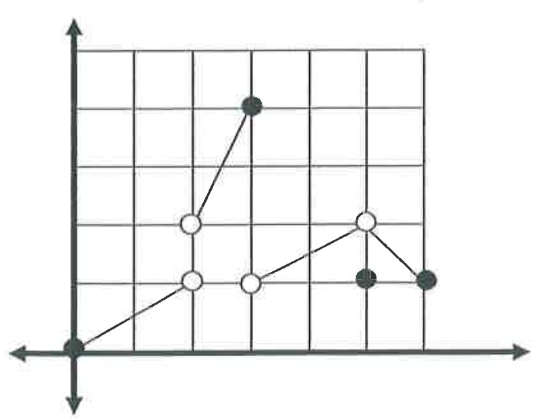
Left hand limit : $\lim_{x \to c^-} f(x)$

- the limit of $f(x)$ as x approaches c from the left

</td></tr>
</table>

2. Find the following limits for the given graph of $f(x)$

- $\lim_{x \to 2^-} f(x) = 1$
- $\lim_{x \to 2^+} f(x) = 2$
- $\lim_{x \to 3^-} f(x) = 4$
- $\lim_{x \to 3^+} f(x) = 1$
- $\lim_{x \to 5^-} f(x) = 2$
- $\lim_{x \to 5^+} f(x) = 2$



For $\lim_{x \to c} f(x)$ to exist, it must equal to both

$\lim_{x \to c^+} f(x)$ and $\lim_{x \to c^-} f(x)$

3. Using problem 2 above, find

a) $\lim_{x \to 2} f(x)$ = DNE

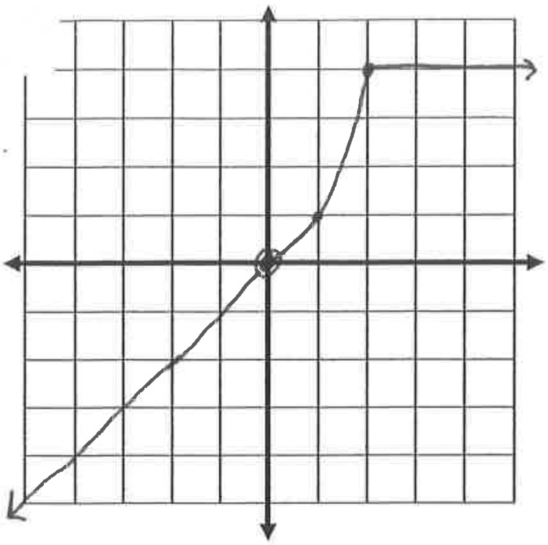
b) $\lim_{x \to 3} f(x)$ = DNE

c) $\lim_{x \to 5} f(x) = 2$

d) at what other points c does $\lim_{x \to c} f(x)$ exist?

in $[0,6)$ except $x=2,3$

4. Graph the piecewise function



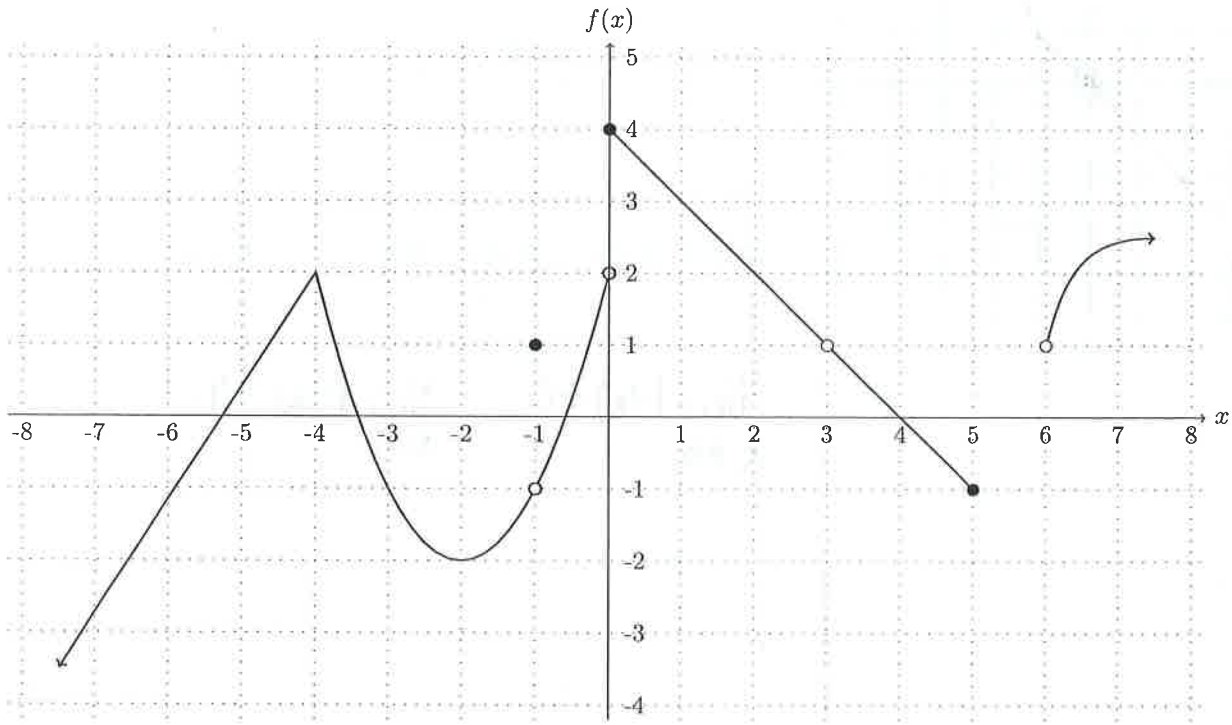
$$f(x)=\begin{cases} x & for \quad x<0 \\ x^2 & for \quad 0\le x\le 2 \\ 4 & for \quad x>2 \end{cases}$$

examine $\lim\limits_{x\to 0}$ and $\lim\limits_{x\to 2}$

$\lim\limits_{x\to 0} f(x)=0$ $\lim\limits_{x\to 2} f(x)=4$

Worksheet – Limits - Graphical Approach.

The goal is to use the graph of $f(x)$ to fill in the table below.

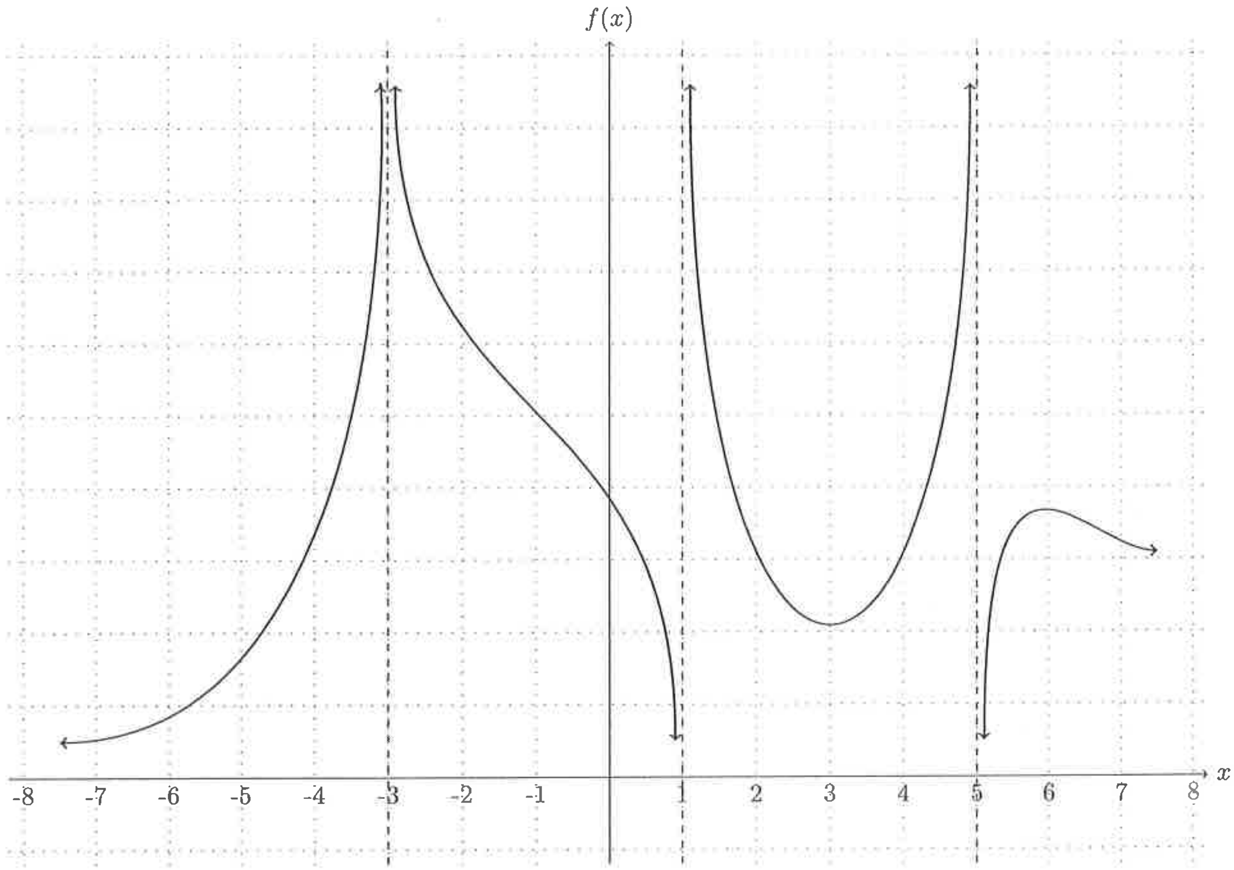


x value	limit from the left	limit from the right	two-sided limit	y value
-6	$\lim_{x\to -6^-} f(x) =$ −1	$\lim_{x\to -6^+} f(x) =$ −1	$\lim_{x\to -6} f(x) =$ −1	$f(-6) =$ −1
-4	$\lim_{x\to -4^-} f(x) =$ 2	$\lim_{x\to -4^+} f(x) =$ 2	$\lim_{x\to -4} f(x) =$ 2	$f(-4) =$ 2
-1	$\lim_{x\to -1^-} f(x) =$ −1	$\lim_{x\to -1^+} f(x) =$ −1	$\lim_{x\to -1} f(x) =$ −1	$f(-1) =$ 1
0	$\lim_{x\to 0^-} f(x) =$ 2	$\lim_{x\to 0^+} f(x) =$ 4	$\lim_{x\to 0} f(x) =$ DNE	$f(0) =$ 4
3	$\lim_{x\to 3^-} f(x) =$ 1	$\lim_{x\to 3^+} f(x) =$ 1	$\lim_{x\to 3} f(x) =$ 1	$f(3) =$ DNE
5	$\lim_{x\to 5^-} f(x) =$ −1	$\lim_{x\to 5^+} f(x) =$ DNE	$\lim_{x\to 5} f(x) =$ DNE	$f(5) =$ −1
6	$\lim_{x\to 6^-} f(x) =$ DNE	$\lim_{x\to 6^+} f(x) =$ 1	$\lim_{x\to 6} f(x) =$ DNE	$f(6) =$ DNE

Key

Worksheet – Infinite Limits - Graphical Approach.

The goal is to use the graph of $f(x)$ to fill in the table below.



x value	limit from the left	limit from the right	two-sided limit	vertical asymptote
-3	$\lim_{x\to -3^-} f(x) = \infty$	$\lim_{x\to -3^+} f(x) = \infty$	$\lim_{x\to -3} f(x) = \infty$	$x = -3$
1	$\lim_{x\to 1^-} f(x) = -\infty$	$\lim_{x\to 1^+} f(x) = \infty$	$\lim_{x\to 1} f(x) =$ DNE	$x = 1$
5	$\lim_{x\to 5^-} f(x) = \infty$	$\lim_{x\to 5^+} f(x) = -\infty$	$\lim_{x\to 5} f(x) =$ DNE	$x = 5$

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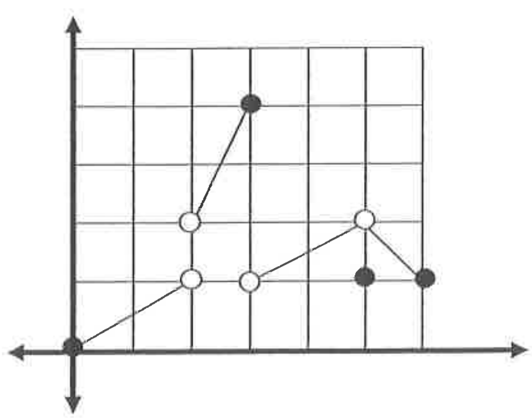

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3. Using problem 2 above, find

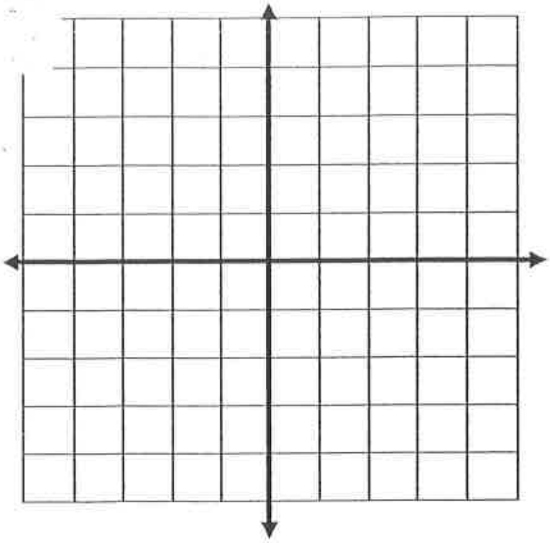
a) $\lim_{x \to 2} f(x)$

b) $\lim_{x \to 3} f(x)$

c) $\lim_{x \to 5} f(x)$

d) at what other points c does $\lim_{x \to c} f(x)$ exist?

4. Graph the piecewise function

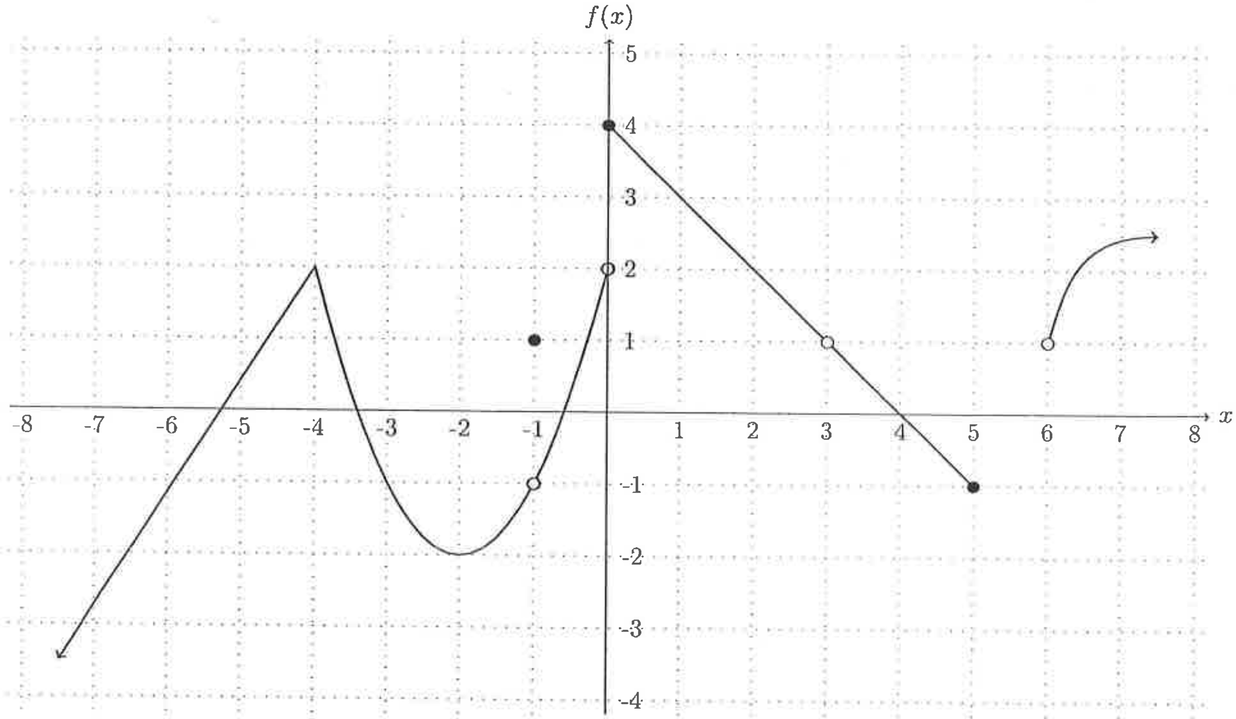


$$f(x) = \begin{cases} x & \text{for} \quad x < 0 \\ x^2 & \text{for} \quad 0 \le x \le 2 \\ 4 & \text{for} \quad x > 2 \end{cases}$$

examine $\lim_{x \to 0}$ and $\lim_{x \to 2}$

Worksheet – Limits - Graphical Approach.

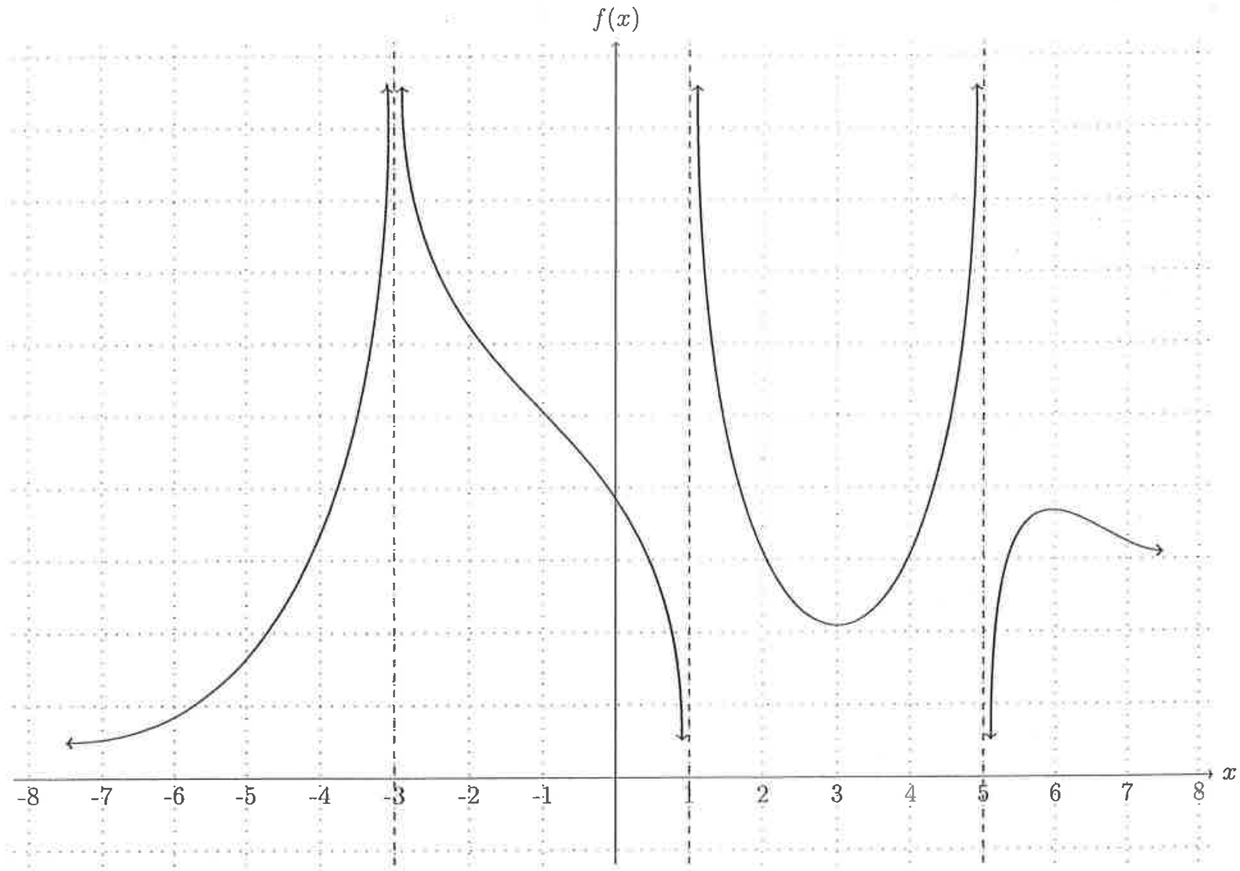
The goal is to use the graph of $f(x)$ to fill in the table below.



x value	limit from the left	limit from the right	two-sided limit	y value
-6	$\lim_{x\to -6^-} f(x) =$	$\lim_{x\to -6^+} f(x) =$	$\lim_{x\to -6} f(x) =$	$f(-6) =$
-4	$\lim_{x\to -4^-} f(x) =$	$\lim_{x\to -4^+} f(x) =$	$\lim_{x\to -4} f(x) =$	$f(-4) =$
-1	$\lim_{x\to -1^-} f(x) =$	$\lim_{x\to -1^+} f(x) =$	$\lim_{x\to -1} f(x) =$	$f(-1) =$
0	$\lim_{x\to 0^-} f(x) =$	$\lim_{x\to 0^+} f(x) =$	$\lim_{x\to 0} f(x) =$	$f(0) =$
3	$\lim_{x\to 3^-} f(x) =$	$\lim_{x\to 3^+} f(x) =$	$\lim_{x\to 3} f(x) =$	$f(3) =$
5	$\lim_{x\to 5^-} f(x) =$	$\lim_{x\to 5^+} f(x) =$	$\lim_{x\to 5} f(x) =$	$f(5) =$
6	$\lim_{x\to 6^-} f(x) =$	$\lim_{x\to 6^+} f(x) =$	$\lim_{x\to 6} f(x) =$	$f(6) =$

Worksheet – Infinite Limits - Graphical Approach.

The goal is to use the graph of $f(x)$ to fill in the table below.



x value	limit from the left	limit from the right	two-sided limit	vertical asymptote
-3	$\lim_{x\to -3^-} f(x) =$	$\lim_{x\to -3^+} f(x) =$	$\lim_{x\to -3} f(x) =$	$x = -3$
1	$\lim_{x\to 1^-} f(x) =$	$\lim_{x\to 1^+} f(x) =$	$\lim_{x\to 1} f(x) =$	$x = 1$
5	$\lim_{x\to 5^-} f(x) =$	$\lim_{x\to 5^+} f(x) =$	$\lim_{x\to 5} f(x) =$	$x = 5$

Limits: A Numeric and Graphic Approach Name: Key

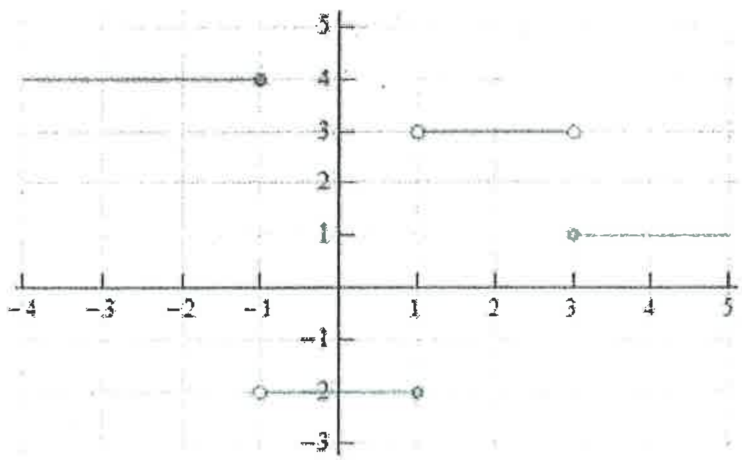
1. For the function $g(x)=\dfrac{x^2+6x+9}{x^2+3x}$ answer each of the following questions. $\dfrac{(x+3)(x+3)}{x(x+3)} = \dfrac{x+3}{3}, x \neq 0, -3$

(a) Evaluate the function the following values of x compute (accurate to at least 8 decimal places).

(i) -2.5	(ii) -2.9	(iii) -2.99	(iv) -2.999	(v) -2.9999
-.2	-.0345	-.0033	-.0003	-.00003
(vi) -3.5	**(vii)** -3.1	**(viii)** -3.01	**(ix)** -3.001	**(x)** -3.0001
.14286	.03226	.00332	.00033	.00003

(b) Use the information from (a) to estimate the value of $\lim_{x\to -3}\dfrac{x^2+6x+9}{x^2+3x}$. $=0$

2. Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$, $\lim_{x\to a^-} f(x)$, $\lim_{x\to a^+} f(x)$, and $\lim_{x\to a} f(x)$. If any of the quantities do not exist clearly explain why.



(a) $a=-1$

$f(-1)=4$

$\lim_{x\to -1^-} f(x)=4$

$\lim_{x\to -1^+} f(x)=-2$

$\lim_{x\to -1} f(x)=$ DNE

(b) $a=1$

$f(1)=-2$

$\lim_{x\to 1^-} = -2$

$\lim_{x\to 1^+} = 3$

$\lim_{x\to 1} =$ DNE

(c) $a=3$

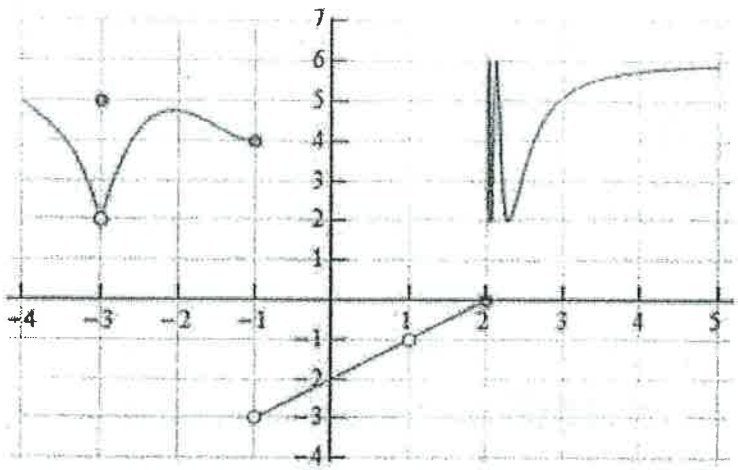
$f(3)=1$

$\lim_{x\to 3^-} = 3$

$\lim_{x\to 3^+} = 1$

$\lim_{x\to 3} =$ DNE

3. Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$, $\lim_{x\to a^-} f(x)$, $\lim_{x\to a^+} f(x)$, and $\lim_{x\to a} f(x)$. If any of the quantities do not exist clearly explain why.



(a) $a=-3$

$f(-3)=5$

$\lim_{x\to -3^-} f(x)=2$

$\lim_{x\to -3^+} f(x)=2$

$\lim_{x\to -3} f(x)=2$

(b) $a=-1$

$f(-1)=4$

$\lim_{x\to -1^-} = 4$

$\lim_{x\to -1^+} = -3$

$\lim_{x\to -1} = DNE$

(c) $a=1$

$f(1)=DNE$

$\lim_{x\to 1^-} = -1$

$\lim_{x\to 1^+} = -1$

$\lim_{x\to 1} = -1$

(d) $a=2$

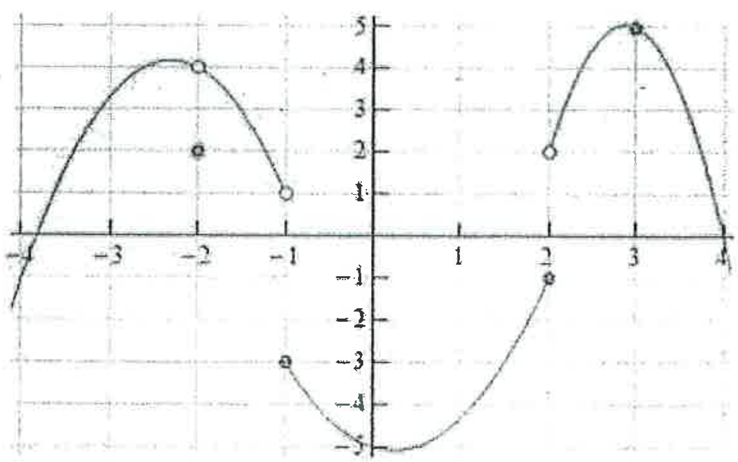
$f(2)=0$

$\lim_{x\to 2^-} = 0$

$\lim_{x\to 2^+} = DNE$

$\lim_{x\to 2} = DNE$

4. Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$ and $\lim_{x\to a} f(x)$. If any of the quantities do not exist clearly explain why.



(a) $a=-2$

$f(-2)=2$

$\lim_{x\to -2^-} f(x)=4$

$\lim_{x\to -2^+} f(x)=4$

$\lim_{x\to 2} f(x)=4$

(b) $a=-1$

$f(-1)=-3$

$\lim_{x\to -1^-} f(x)=1$

$\lim_{x\to -1^+} f(x)=-3$

$\lim_{x\to -1} f(x)=DNE$

(c) $a=2$

$f(2)=-1$

$\lim_{x\to 2^-} = -1$

$\lim_{x\to 2^+} = 2$

$\lim_{x\to 2} = DNE$

(d) $a=3$

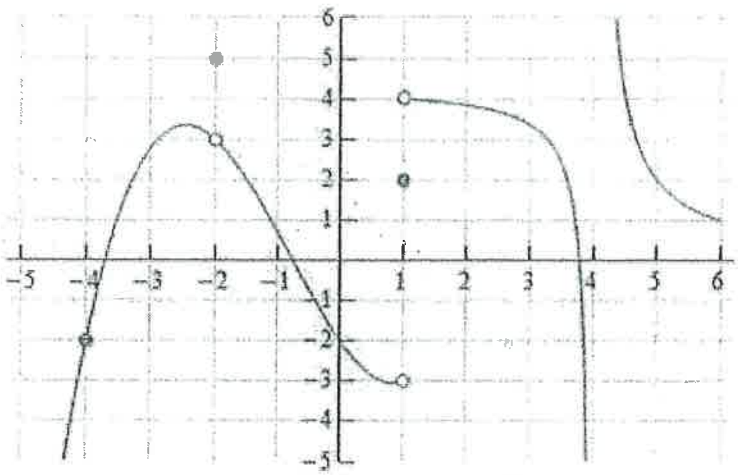
$f(3)=5$

$\lim_{x\to 3^-} = 5$

$\lim_{x\to 3^+} = 5$

$\lim_{x\to 3} = 5$

5.Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$ and $\lim_{x\to a} f(x)$. If any of the quantities do not exist clearly explain why.



(a) $a=-4$

$f(-4)=-2$

$\lim_{x\to -4} f(x)=-2$

(b) $a=-2$

$f(-2)=5$

$\lim_{x\to -2} f(x)=3$

(c) $a=1$

$f(1)=2$

$\lim_{x\to 1} f(x)=\text{DNE}$

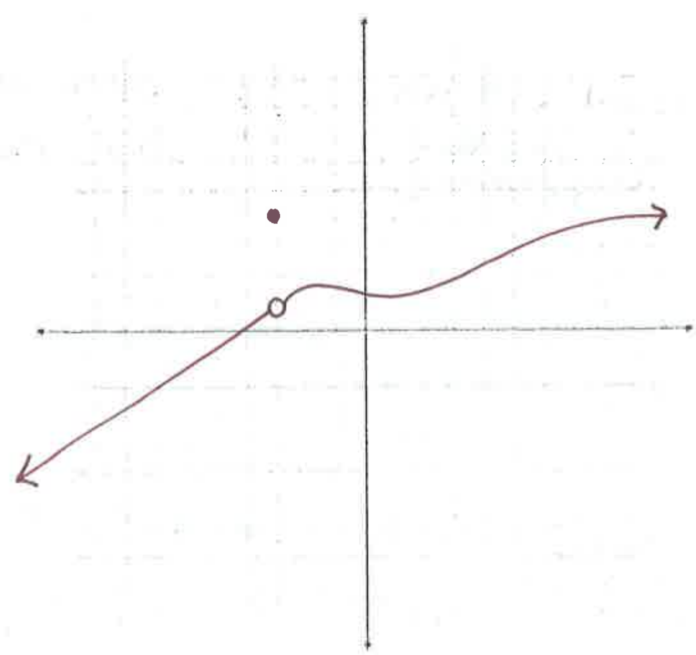
(d) $a=4$

$f(4)=\text{DNE}$

$\lim_{x\to 4} f(x)=\text{DNE}$

6. Sketch a graph of a function that satisfies each of the following conditions.

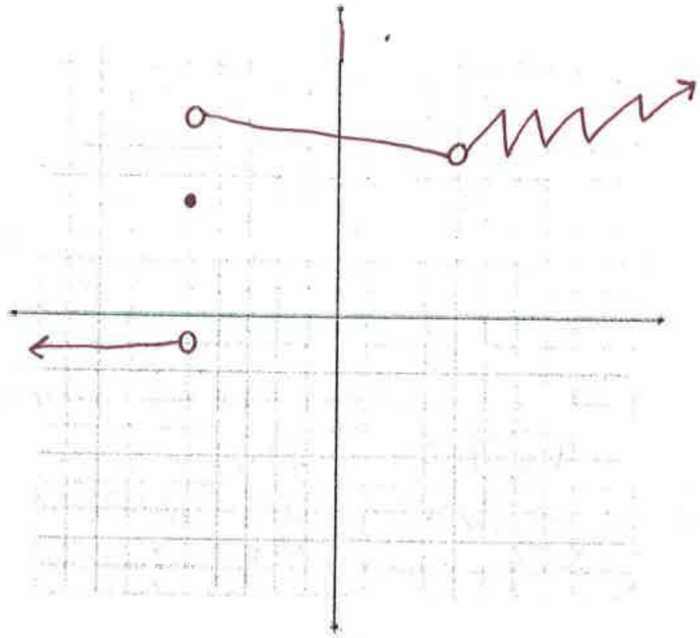
$\lim_{x\to -3^-} f(x)=1$ $\quad$ $\lim_{x\to -3^+} f(x)=1$ $\quad$ $f(-3)=4$



7. Sketch a graph of a function that satisfies each of the following conditions.

$\lim_{x \to -5^-} f(x) = -1$ $\lim_{x \to -5^+} f(x) = 7$ $f(-5) = 4$

$\lim_{x \to 4} f(x) = 6$ $f(4)$ does not exist



8. Explain in your own words what each of the following equations mean.

$\lim_{x \to 8^-} f(x) = 3$ $\lim_{x \to 8^+} f(x) = -1$

as x approaches 8 from left, the y-value is approaching 3

as x approaches 8 from right, the y-value is approaching -1

9. Explain in your own words what the following equation means.

$$\lim_{x \to 12} f(x) = 6$$

As x approaches 12 from both sides, the function is approaching y=6

10. Suppose we know that $f(6) = -53$. If possible, determine the value of $\lim_{x \to 6^-} f(x)$ and the value of $\lim_{x \to 6^+} f(x)$. If it is not possible to determine one or both of these values explain why not.

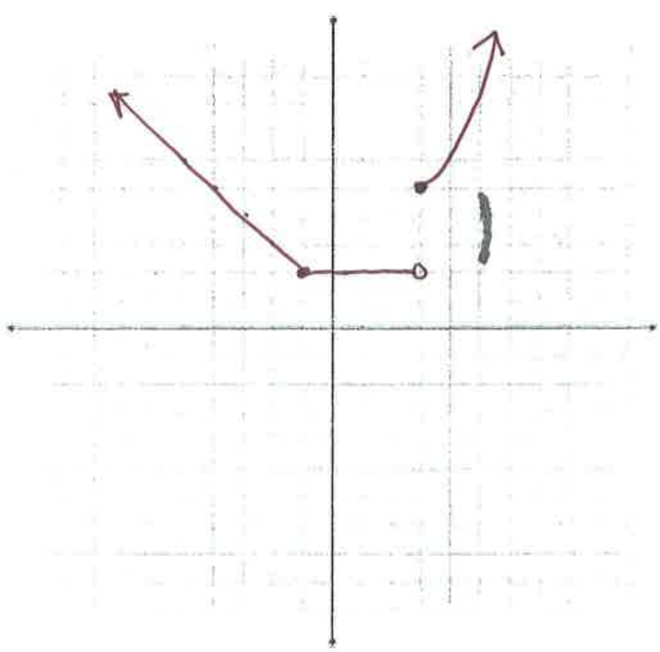
Not possible, there could be a hole at f(6)

11. Is it possible to have $\lim_{x \to 1} f(x) = -23$ and $f(1) = 107$? Explain your answer.

Yes, the function curve could approach -23 on both sides and have a hole, where f(1)=107 can be a single point on the graph

12. Graph the piece-wise function. Note the interval(s) where the limit is defined.

$$f(x) = \begin{cases} -x+1, & x \le -1 \\ 2, & -1 < x < 3 \\ x^2 - 4, & x \ge 3. \end{cases}$$



$\lim_{x \to c} f(c)$ is defined everywhere except $c = 3$

Limits: A Numeric and Graphic Approach Name: ______________________

1. For the function $g(x)=\dfrac{x^2+6x+9}{x^2+3x}$ answer each of the following questions.

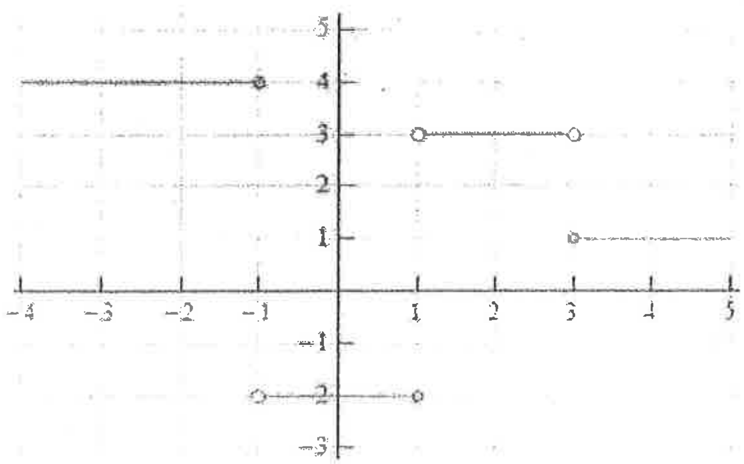
(a) Evaluate the function the following values of x compute (accurate to at least 8 decimal places).

(i) -2.5 **(ii)** -2.9 **(iii)** -2.99 **(iv)** -2.999 **(v)** -2.9999

(vi) -3.5 **(vii)** -3.1 **(viii)** -3.01 **(ix)** -3.001 **(x)** -3.0001

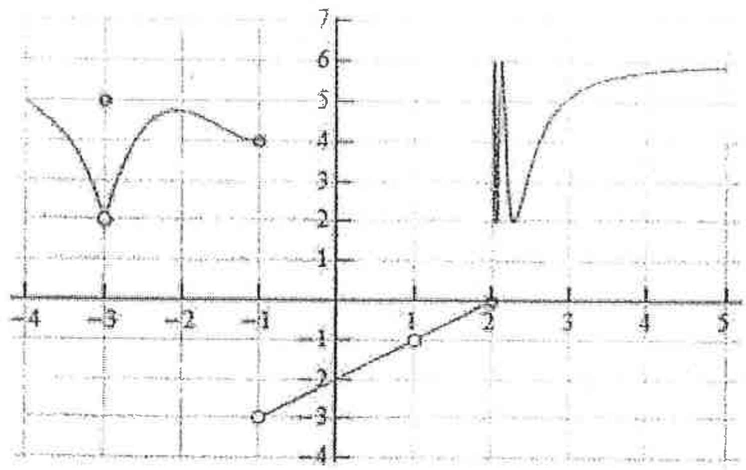
(b) Use the information from **(a)** to estimate the value of $\lim\limits_{x\to -3}\dfrac{x^2+6x+9}{x^2+3x}$.

2. Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$, $\lim\limits_{x\to a^-} f(x)$, $\lim\limits_{x\to a^+} f(x)$, and $\lim\limits_{x\to a} f(x)$. If any of the quantities do not exist clearly explain why.



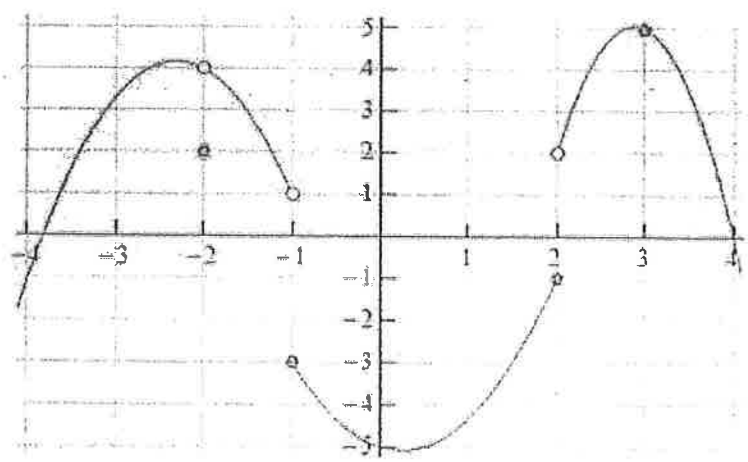
(a) $a=-1$ (b) $a=1$ (c) $a=3$

3. Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$, $\lim_{x \to a^-} f(x)$, $\lim_{x \to a^+} f(x)$, and $\lim_{x \to a} f(x)$. If any of the quantities do not exist clearly explain why.



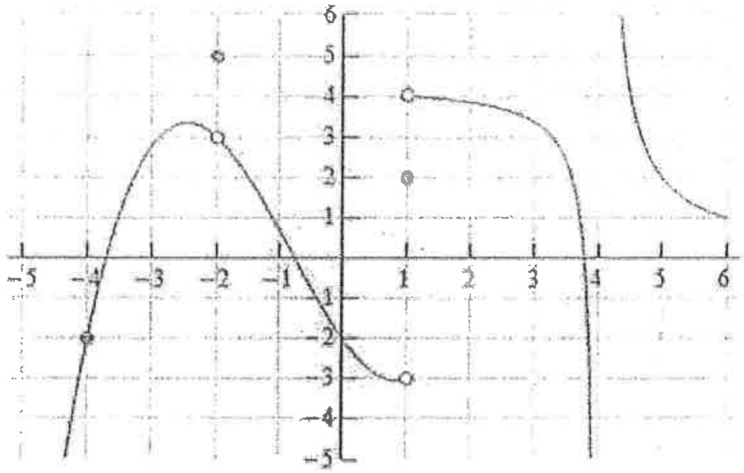
(a) $a=-3$ (b) $a=-1$ (c) $a=1$ (d) $a=2$

4. Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$ and $\lim_{x \to a} f(x)$. If any of the quantities do not exist clearly explain why.



(a) $a=-2$ (b) $a=-1$ (c) $a=2$ (d) $a=3$

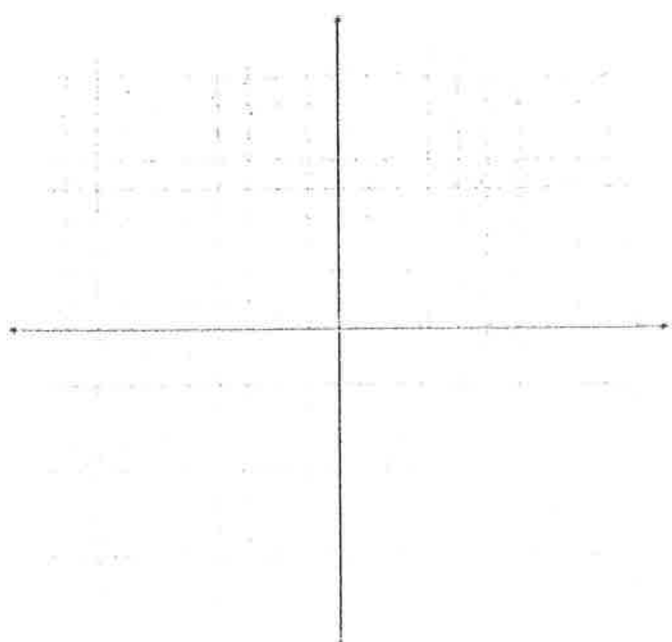
5.Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$ and $\lim_{x \to a} f(x)$. If any of the quantities do not exist clearly explain why.



(a) $a = -4$ **(b)** $a = -2$ **(c)** $a = 1$ **(d)** $a = 4$

6. Sketch a graph of a function that satisfies each of the following conditions.

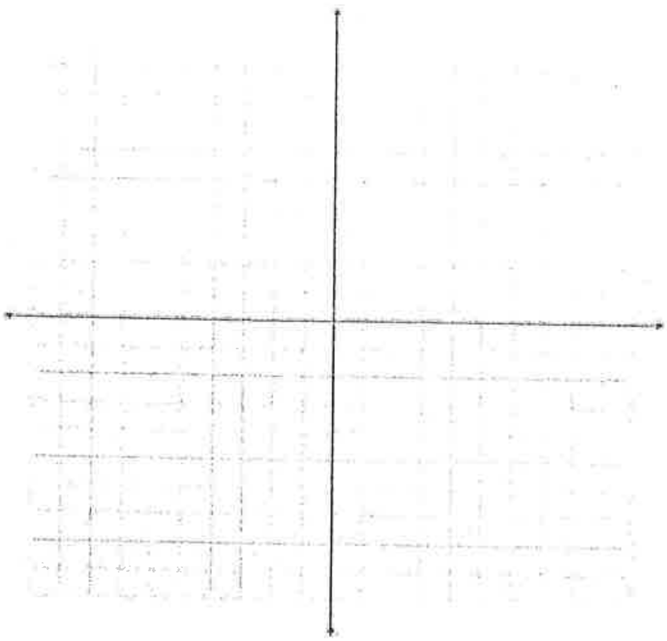
$\lim_{x \to -3^-} f(x) = 1$ $\lim_{x \to -3^+} f(x) = 1$ $f(-3) = 4$



7. Sketch a graph of a function that satisfies each of the following conditions.

$\lim_{x \to -5^-} f(x) = -1$ $\lim_{x \to -5^+} f(x) = 7$ $f(-5) = 4$

$\lim_{x \to 4} f(x) = 6$ $f(4)$ does not exist



8. Explain in your own words what each of the following equations mean.

$\lim_{x \to 8^-} f(x) = 3$ $\lim_{x \to 8^+} f(x) = -1$

9. Explain in your own words what the following equation means.

$$\lim_{x \to 12} f(x) = 6$$

10. Suppose we know that $f(6) = -53$. If possible, determine the value of $\lim_{x \to 6^-} f(x)$ and the value of $\lim_{x \to 6^+} f(x)$. If it is not possible to determine one or both of these values explain why not.

11. Is it possible to have $\lim_{x \to 1} f(x) = -23$ and $f(1) = 107$? Explain your answer.

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