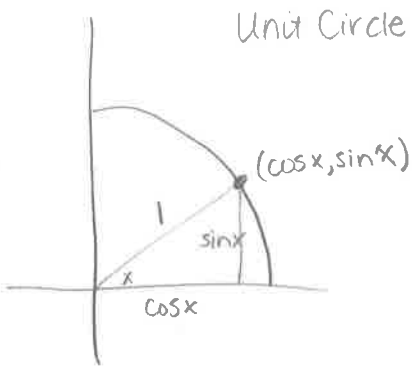
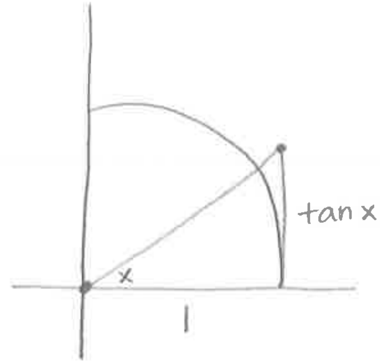




Prove:

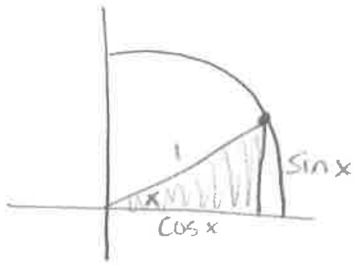
$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

Using squeeze theorem

Similar Δs

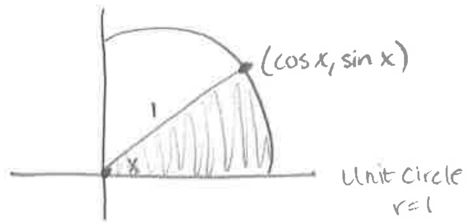
$$\frac{\sin x}{?} = \frac{\cos x}{1} \Rightarrow \sin x = ? \cdot \cos x \Rightarrow ? = \tan x$$

Triangle 1:



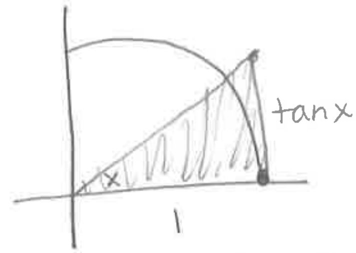
$$A = \frac{1}{2}(\cos x)(\sin x)$$

Sector:



$$A = \frac{x}{2\pi} \cdot \pi = \frac{x}{2}$$

Triangle 2:



$$A = \frac{1}{2}(\tan x) = \frac{1}{2}\frac{\sin x}{\cos x}$$

$$A_{T_1} \leq A_S \leq A_{T_2}$$

$$\left(\frac{1}{2}(\cos x)(\sin x) \leq \frac{x}{2} \leq \frac{\sin x}{2\cos x}\right) \cdot \frac{2}{\sin x}$$

$$\cos x \leq \frac{x}{\sin x} \leq \frac{1}{\cos x}$$

$$\lim_{x \to 0} \cos x = 1 \qquad \lim_{x \to 0} \frac{1}{\cos x} = 1$$

$$1 \leq \frac{x}{\sin x} \leq 1$$ reciprocal

$$1 \leq \frac{\sin x}{x} \leq 1 \qquad \therefore \lim_{x \to 0} \frac{\sin x}{x} = 1$$