Key

Theorem 1.6 Limits of Trigonometric Functions

1. $\lim_{x\to c} \sin x = \sin c$	3. $\lim_{x\to c} \tan x = \tan c$	5. $\lim_{x\to c} \sec x = \sec c$
2. $\lim_{x\to c} \cos x = \cos c$	4. $\lim_{x\to c} \cot x = \cot c$	6. $\lim_{x\to c} \csc x = \csc c$

Example: Find each limit

(a) $\lim_{x\to 1}(-x^2+1) = -1+1=0$

(b) $\lim_{x\to 4}\sqrt[3]{x+4} = \sqrt[3]{8}=2$

(c) $\lim_{x\to 3}\frac{\sqrt{x+1}}{x-4} = \frac{\sqrt{3+1}}{3-4} = \frac{2}{-1} = -2$

(d) $\lim_{x\to 7}\sec\left(\frac{\pi x}{6}\right) = \sec\frac{7\pi}{6} = \frac{1}{\cos\left(\frac{7\pi}{6}\right)} = -\frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{-2\sqrt{3}}{3}$

Example: Use the given information to evaluate the limits: $\lim_{x\to c} f(x)=2$ and $\lim_{x\to c} g(x)=3$

(a) $\lim_{x\to c}[5g(x)]$ $5(3)=15$

(b) $\lim_{x\to c}[f(x)+g(x)]$ $2+3=5$

(c) $\lim_{x\to c}[f(x)g(x)]$ $(2)(3)=6$

(d) $\lim_{x\to c}\frac{f(x)}{g(x)}$ $\frac{2}{3}$

A Strategy for Finding Limits

If a limit cannot be found using direct substitution, then we will use the following theorem and some other techniques to evaluate the limit.

Theorem 1.7 Functions That Agree at All But One Point

Let c be a real number and let $f(x)=g(x)$ for all $x \neq c$ in an open interval containing c. If the limit of $g(x)$ as x approaches c exists, then the limit of $f(x)$ also exists and

$$\lim_{x\to c} f(x) = \lim_{x\to c} g(x).$$

♫: Keep in mind that some functions do not have limits.

If direct substitution yields the meaningless result $\frac{0}{0}$, then you cannot determine the limit in this form. The expression that yields this result is called an **indeterminate form.** When you encounter this form, you must rewrite the fraction so that the new denominator does not have 0 as its limit. One way to do this is to *cancel like factors*, and a second way is to *rationalize the numerator*.

Example: Find the limit: $\lim_{x\to -1} \frac{2x^2-x-3}{x+1} = \frac{0}{0}$

$$\frac{(x+1)(2x-3)}{(x+1)} = 2x-3$$

$$\lim_{x\to -1} 2x-3 = -5$$

Example: Find the limit (if it exists): $\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} = \frac{0}{0}$

$$\frac{(\sqrt{x+1}-2)}{(x-3)}\frac{(\sqrt{x+1}+2)}{(\sqrt{x+1}+2)} = \frac{x+1-4}{(x-3)(\sqrt{x+1}+2)} = \frac{(x-3)}{(x-3)(\sqrt{x+1}+2)}$$

$$\frac{1}{\sqrt{x+1}+2} = \frac{1}{4}$$

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5. $\lim_{x \to c} \sec x = \sec c$
6. $\lim_{x \to c} \csc x = \csc c$

Example: Find each limit

(a) $\lim_{x \to 1} \left(-x^2 + 1\right)$

(b) $\lim_{x \to 4} \sqrt[3]{x+4}$

(c) $\lim_{x \to 3} \frac{\sqrt{x+1}}{x-4}$

(d) $\lim_{x \to 7} \sec\left(\frac{\pi x}{6}\right)$

Example: Use the given information to evaluate the limits: $\lim_{x \to c} f(x) = 2$ and $\lim_{x \to c} g(x) = 3$

(a) $\lim_{x \to c} \left[5g(x)\right]$

(b) $\lim_{x \to c} \left[f(x) + g(x)\right]$

(c) $\lim_{x \to c} \left[f(x)g(x)\right]$

(d) $\lim_{x \to c} \frac{f(x)}{g(x)}$

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Example: Find the limit: $\lim_{x \to -1} \frac{2x^2 - x - 3}{x+1}$

Example: Find the limit (if it exists): $\lim_{x \to 3} \frac{\sqrt{x+1} - 2}{x-3}$