5e - do not rationalize!

19 - Should be a t

Extra practice Key

~~(d) $h(6z)$~~ ~~(e) $h(1-3y)$~~ ~~(f) $h(y+k)$~~

3. $g(t)=\dfrac{t+5}{1-t}$

   (a) $g(0) = 5$ (b) $g(4)$ -3 (c) $g(-7)$ $-\frac{1}{4}$

   (d) $g(x^2-5)$ (e) $g(t+h)$ (f) $g(4\sqrt{t}+9)$

d) $\dfrac{x^2-5+5}{1-(x^2-5)} = \dfrac{x^2}{-x^2-4}$

e) $\dfrac{t+h+5}{1-t-h}$

f) $\dfrac{4\sqrt{t}+9+5}{1-(4\sqrt{t}+9)} = \dfrac{4\sqrt{t}+14}{-4\sqrt{t}-8}$

4. $f(z)=\sqrt{4z+5}$

   (a) $f(0)$ $\sqrt{5}$ (b) $f(-1)$ 1 (c) $f(-2)$ no real answer

   (d) $h(5-12y)$ (e) $f(2z^2+8)$ (f) $f(z+h)$

d) $\sqrt{4(5-12y)+5} = \sqrt{25-48y}$

e) $\sqrt{4(2z^2+8)+5} = \sqrt{8z^2+32+5}$ $\sqrt{8z^2+37}$

f) $\sqrt{4(z+h)+5} = \sqrt{4z+4h+5}$

5. $z(x)=\dfrac{\sqrt{x^2+9}}{4x+8}$

   (a) $z(4)$ $\frac{5}{24}$ (b) $z(-4)$ $-\frac{5}{8}$ (c) $z(1)$ $\frac{\sqrt{10}}{12}$

   (d) $z(2-7x)$ (e) $z(\sqrt{3x+4})$ (f) $z(x+h)$

d) $\dfrac{\sqrt{(2-7x)^2+9}}{4(2-7x)+8} = \dfrac{\sqrt{4-28x+49x^2+9}}{8-7x+8} = \dfrac{\sqrt{13-28x+49x^2}}{16-7x}$

e) $\dfrac{\sqrt{(\sqrt{3x+4})^2+9}}{4\sqrt{3x+4}+8} = \dfrac{\sqrt{3x+4+9}}{4\sqrt{3x+4}+8} = \dfrac{\sqrt{3x+13}}{4\sqrt{3x+4}+8}$

f) $\dfrac{\sqrt{(x+h)^2+9}}{4(x+h)+8} = \dfrac{\sqrt{x^2+2hx+h^2+9}}{4x+4h+8}$

6. $Y(t)=\sqrt{3-t}-\dfrac{t}{2t+5}$

   (a) $Y(0)$ $\sqrt{3}$ (b) $Y(7)$ non real answer (c) $Y(-4)$ $\sqrt{7}+\frac{4}{13}$

   (d) $Y(5-t)$ (e) $Y(t^2-10)$ (f) $Y(6t-t^2)$

d) $\sqrt{3-(5-t)} - \dfrac{(5-t)}{2(5-t)+5}$ $\sqrt{-2+t} - \dfrac{5-t}{15-2t}$

e) $\sqrt{3-(t^2-10)} - \dfrac{t^2-10}{2(t^2-10)+5}$ $\sqrt{13-t^2} - \dfrac{t^2-10}{2t^2-15}$

f) $\sqrt{3-(6t-t^2)} - \dfrac{6t-t^2}{2(6t-t^2)+5}$ $\boxed{\sqrt{3-6t+t^2} - \dfrac{6t-t^2}{12t-2t^2+5}}$

The **difference quotient** of a function $f(x)$ is defined to be,

$$\frac{f(x+h)-f(x)}{h}$$

For problems 7 – 13 compute the difference quotient of the given function.

7. $Q(t)=4-7t$

$$\frac{4-7(t+h)-(4-7t)}{h} = \frac{\cancel{4}\cancel{-7t}-7h\cancel{-4}\cancel{+7t}}{h} = -7$$

8. $g(t)=42$

$$\frac{42-42}{h} = 0$$

9. $H(x)=2x^2+9$

$$\frac{2(x+h)^2+9-(2x^2+9)}{h} = \frac{2(x^2+2hx+h^2)+9-2x^2-9}{h} = \frac{\cancel{2x^2}+4hx+2h^2\cancel{+9}\cancel{-2x^2}\cancel{-9}}{h} = \frac{2h(2x+h)}{h} = 2(2x+h)$$

10. $z(y)=3-8y-y^2$

$$\frac{3-8(y+h)-(y+h)^2-3+8y+y^2}{h} = \frac{\cancel{3}\cancel{-8y}-8h\cancel{-y^2}-2hy-h^2\cancel{-3}\cancel{+8y}\cancel{+y^2}}{h}$$

$$\frac{\cancel{h}(-8-2y-h)}{\cancel{h}} = -8-2y-h$$

$\frac{\sqrt{4+3(x+h)}-\sqrt{4+3x}}{h} = \frac{\sqrt{4+3x+3h}-\sqrt{4+3x}}{h}$

11. $g(z)=\sqrt{4+3z}$

12. $y(x)=\frac{-4}{1-2x}$ $\quad \frac{\frac{-4}{1-2(x+h)}-\frac{-4}{1-2x}}{h} = \frac{\frac{-4}{1-2x-2h}+\frac{4}{1-2x}}{h} = \frac{\cancel{-4+8x}+4\cancel{-8x}-8h}{(1-2x-2h)(1-2x)}\cdot\frac{1}{h} = \frac{-8}{(1-2x-2h)(1-2x)}$

13. $f(t)=\frac{t^2}{t+7}$ $\quad \frac{\frac{(t+h)^2}{(t+h)+7}-\frac{t^2}{t+7}}{h} = \frac{\frac{t^2+2ht+h^2}{t+h+7}-\frac{t^2}{t+7}}{h} = \frac{\cancel{t^3}+2ht^2+h^2t+\cancel{7t^2}+14ht+7h^2-\cancel{t^3}-ht^2-\cancel{7t^2}}{(t+h+7)(t+7)}\cdot\frac{1}{h} =$

$\frac{ht^2+h^2t+14ht+7h^2}{h(t+h+7)(t+7)} = \frac{t^2+ht+14t+7h}{(t+h+7)(t+7)}$

For problems 14 – 21 determine all the roots of the given function.

F 14. $y(t)=40+3t-t^2$ $\quad -(t^2-3t-40)=0$ $\quad -(t-8)(t+5)=0$ $\quad t=8$ $\quad t=-5$

-24 $\quad -8+3$

F 15. $f(x)=6x^4-5x^3-4x^2$ $\quad x^2(6x^2-5x-4)=0$ $\quad x^2(6x^2-8x+3x-4)=0$ $\quad x^2(2x+1)(3x-4)=0$ $\quad x=0,\ x=-1/2\ \ x=4/3$

Q 16. $Z(p)=6-11p-p^2$ $\quad \frac{+11\pm\sqrt{121-4(-1)(6)}}{2(-1)}=\frac{11\pm\sqrt{145}}{-2}$

Q 17. $h(y)=4y^6+10y^5+y^4$ $\quad y^4(4y^2+10y+1)$ $\quad \frac{-10\pm\sqrt{100-4(4)(1)}}{2(4)}=\frac{-10\pm\sqrt{84}}{8}=\frac{-10\pm2\sqrt{21}}{8}$ $\quad y=0$ $\quad y=\frac{-5\pm\sqrt{21}}{4}$

F 18. $g(z)=z^7+6z^4-16z$ $\quad z(z^6+6z^3-16)=0$ $\quad z(z^3+8)(z^3-2)=0$ $\quad z=0\ \ z=-2\ \ z=\sqrt[3]{2}$

should be +

F 19. $f(t)=t^{\frac{1}{2}}-8t^{\frac{1}{4}}+15$ $\quad (t^{1/4}-5)(t^{1/4}-3)=0$ $\quad t=81\ \ t=625$

20. $h(w)=\frac{w}{4w+5}+\frac{3w}{w-8}$ $\quad \frac{w^2-8w+12w^2+15w}{(4w+5)(w-8)}=\frac{11w^2+7w}{(4w+5)(w-8)}=\frac{w(11w+7)}{(4w+5)(w-8)}=0$ $\quad w=0$ $\quad w=-7/11$

roots are when the function (numerator) = zero

Q 21. $g(w)=\frac{w}{w+3}-\frac{w+2}{4w-1}$ $\quad \frac{4w^2-w-(w^2+5w+6)}{(w+3)(4w-1)}=\frac{3w^2-6w-6}{(w+3)(4w-1)}=\frac{3(w^2-2w-2)}{(w+3)(4w-1)}$

$\frac{+2\pm\sqrt{4-4(1)(-2)}}{2(1)}=\frac{2\pm2\sqrt3}{2}=1\pm\sqrt3$

For problems 22 – 30 find the domain and range of the given function.

22. $f(x)=x^2-8x+3$ $\quad$ D: $\mathbb{R}$ $\quad$ R: $[-13,\infty)$ $\quad \frac{+8}{2(1)}=\frac{8}{2}=4$ $\quad 4^2-8(4)+3=16-32+3=-13$

vertex $(4,-13)$

23. $z(w)=4-7w-w^2$ $\quad$ D: $\mathbb{R}$ $\quad$ R: $(-\infty,\frac{-131}{4}]$ $\quad \frac{+7}{2(-1)}=\frac{-7}{2}$ $\quad -(\frac{7}{2})^2-7(\frac{7}{2})+4=\frac{-49}{4}-\frac{98}{4}+\frac{16}{4}=\frac{-131}{4}$

$-w^2-7w+4$

vertex $(\frac{-7}{2},\frac{-131}{4})$

24. $g(t)=3t^2+2t-3$ $\quad$ D: $\mathbb{R}$ $\quad$ R: $[-4,\infty)$

$\frac{-2}{2(3)}=\frac{-1}{3}$ $\quad 3(-\frac{1}{3})^2+2(-\frac{1}{3})-3=\frac{-1}{3}+\frac{-2}{3}-\frac{9}{3}=\frac{-12}{3}=-4$

vertex $(-\frac{1}{3},-4)$