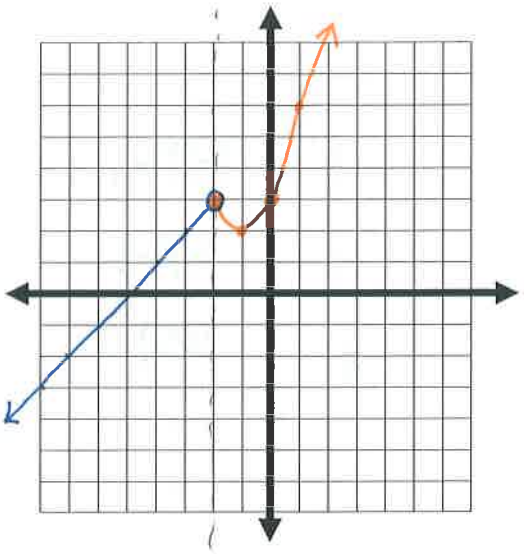
Piecewise Functions *Name:* Key

Part I. Carefully graph each of the following. Identify whether or not the graph is a function. Then, evaluate the graph at any specified domain value. You may use your calculators to help you graph, but you must sketch it carefully on the grid!

1. $f(x) = \begin{cases} x+5 & x < -2 \\ x^2+2x+3 & x \geq -2 \end{cases}$



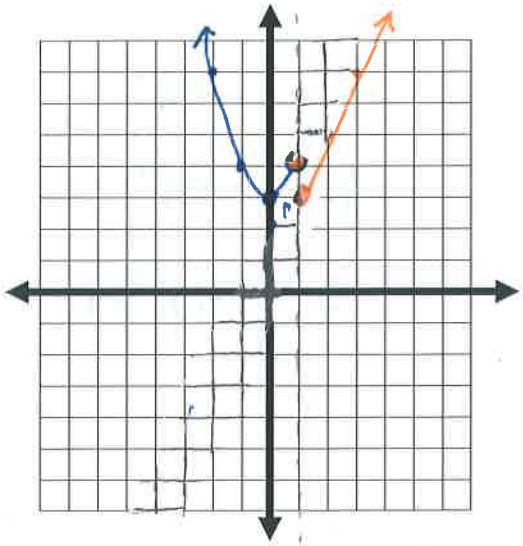
Function? (Yes) or No

$f(3) = 18$ $(3)^2+2(3)+3$

$f(-4) = 1$ $-4+5$

$f(-2) = 3$ $(-2)^2+2(-2)+3$

2. $f(x) = \begin{cases} 2x+1 & x \geq 1 \\ x^2+3 & x < 1 \end{cases}$



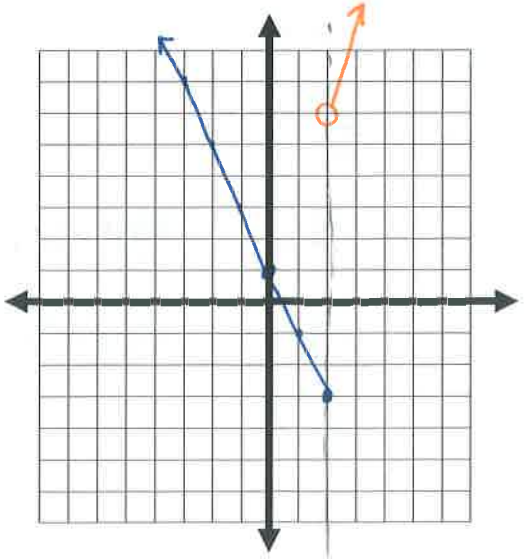
Function? (Yes) or (No)

$f(-2) = 7$ $(-2)^2+3$

$f(6) = 13$ $2(6)+1$

$f(1) = 3$ $2(1)+1$

3. $f(x) = \begin{cases} -2x+1 & x \leq 2 \\ 5x-4 & x > 2 \end{cases}$



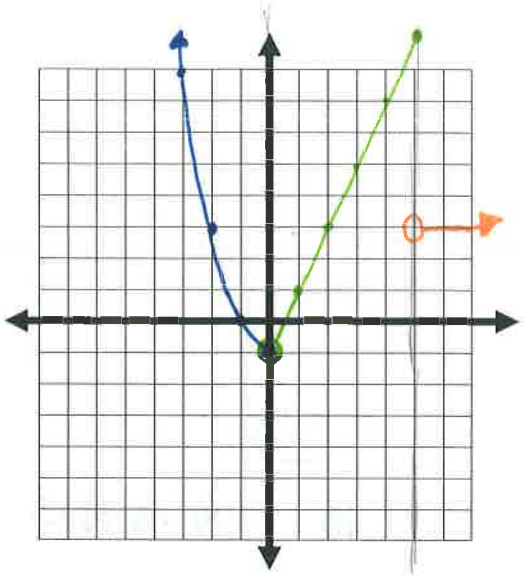
Function? (Yes) or (No)

$f(-4) = 9$ $-2(-4)+1$

$f(8) = 36$ $5(8)-4$

$f(2) = -3$ $-2(2)+1$

4. $f(x) = \begin{cases} x^2 - 1 & x \le 0 \\ 2x - 1 & 0 < x \le 5 \\ 3 & x > 5 \end{cases}$



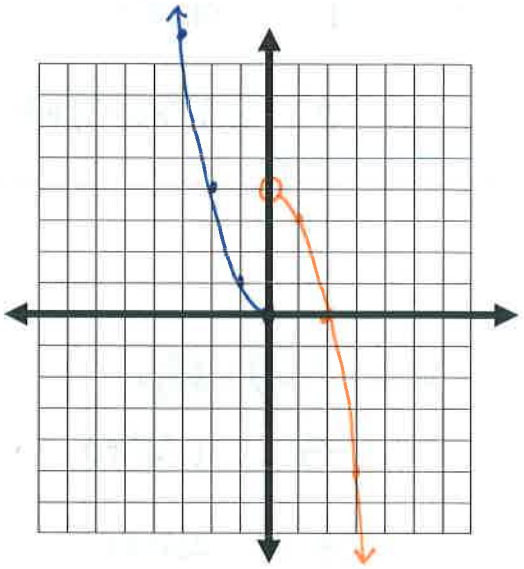
Function? (Yes) or No

$f(-2) = 3$ $(-2)^2 - 1$

$f(0) = -1$ $(0)^2 - 1$

$f(5) = 9$ $2(5) - 1$

5. $f(x) = \begin{cases} x^2 & x \le 0 \\ -x^2 + 4 & x > 0 \end{cases}$



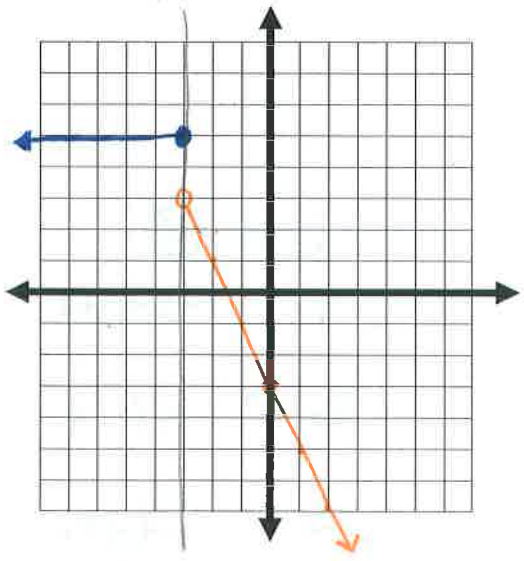
Function? (Yes) or No

$f(-4) = 16$ $(-4)^2$

$f(0) = 0$ $(0)^2$

$f(3) = -5$ $-(3)^2 + 4$

6. $f(x) = \begin{cases} 5 & x \le -3 \\ -2x - 3 & x > -3 \end{cases}$



Function? (Yes) or No

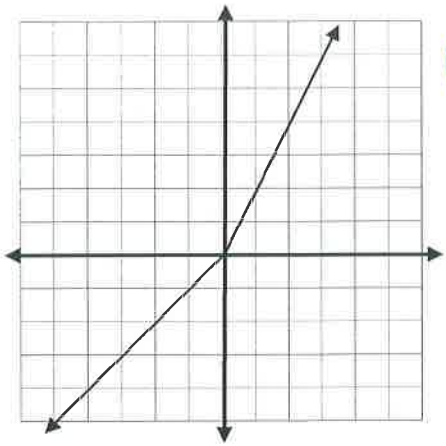
$f(-4) = 5$

$f(0) = -3$ $-2(0) - 3$

$f(3) = -9$ $-2(3) - 3$

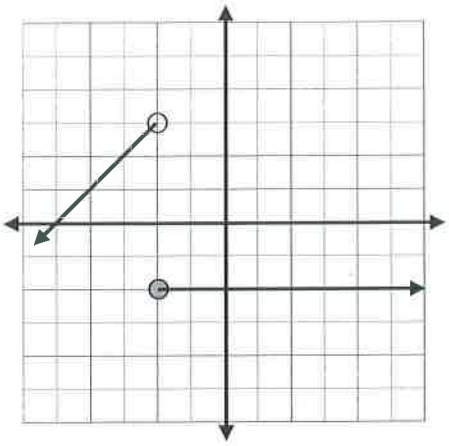
Part II. Write equations for the piecewise functions whose graphs are shown below. Assume that the units are 1 for every tic marc.

7.



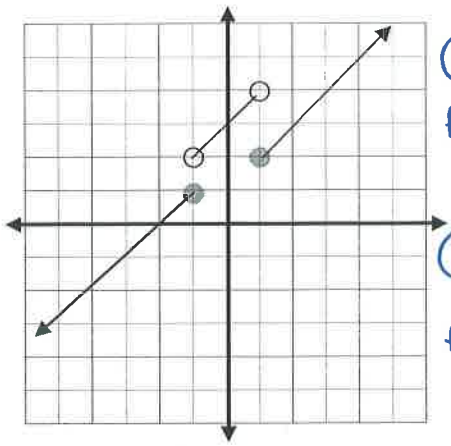
(7) $f(x) = \begin{cases} x & x<0 \\ 2x & x \geq 0 \end{cases}$

8.



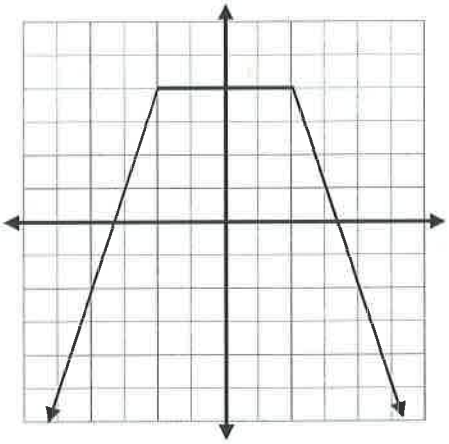
(8) $f(x) = \begin{cases} x+5 & x<-2 \\ -2 & x \geq -2 \end{cases}$

9.



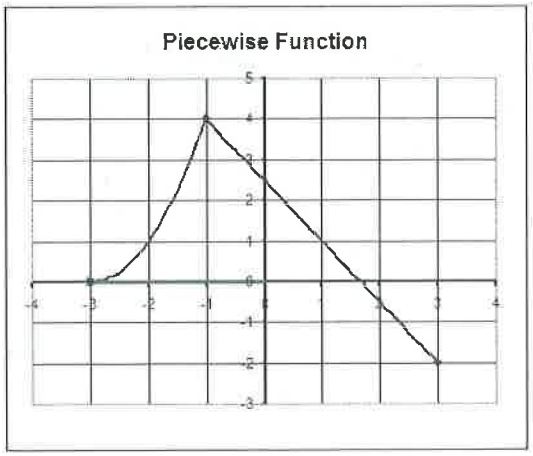
(9) $f(x) = \begin{cases} x+2 & x \leq -1 \\ x+3 & -1<x<1 \\ x+1 & x \geq 1 \end{cases}$

10.



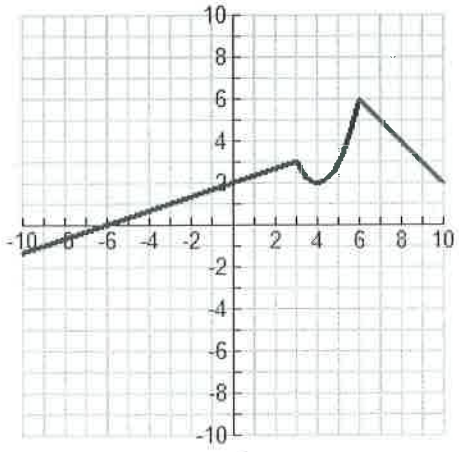
(10) $f(x) = \begin{cases} 3x+10 & x<-2 \\ 4 & -2 \leq x \leq 2 \\ -3x+10 & x>2 \end{cases}$

11.



(11) $f(x) = \begin{cases} (x+3)^2 & -3 \leq x \leq -1 \\ -\frac{3}{2}x + \frac{5}{2} & x>-1 \end{cases}$

12.



(12) $f(x) = \begin{cases} \frac{1}{3}x+2 & x<3 \\ (x-4)^2+2 & 3 \leq x \leq 6 \\ -x+12 & x>6 \end{cases}$

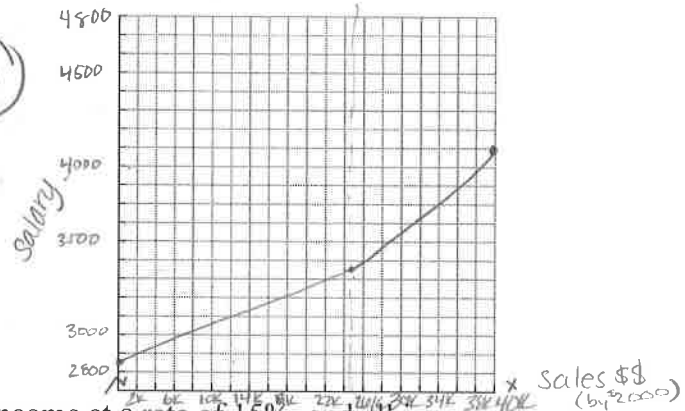
1. An air conditioning salesperson receives a base salary of $2850 per month plus a commission. The commission is 2% of the sales up to and including $25,000 for the month and 5% of the sales over $25,000 for the month.
   a. Write a piecewise function that relates the salesperson total monthly income based off or his/her sales for the month.
   b. Sketch an accurate graph of this piecewise function
   c. Determine the salesperson's monthly income if his/her sales were $43,000 for the month.

Base = 2850
2% = 2850 + .02x
5% = 2850 + 500 + .05x = 3350 + .05x

(a) $f(x) = \begin{cases} 2850 + .02x & 0 \le x \le 25000 \\ 3350 + .05(x - 25000) & x > 25000 \end{cases}$

(c) $3350 + .05(43000 - 25000) = \$4250$

(b)



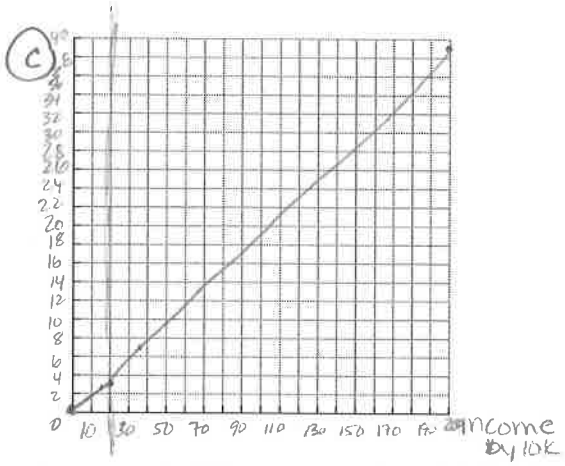
2. A certain country taxes the first $20,000 of an individual's income at a rate of 15%, and all income over $20,000 is taxed at 20%.
   a. Al makes $16,000. Betty makes $36,000. How much is each taxed?
   b. Write a piecewise function T that specifies the total tax on an income of x dollars.
   c. Make a graph of T. Be sure to plot the points from part a!
   d. Catina is taxed $5000. What is her income?

(a) Al $.15 \times 16000 = \$2400$
Betty $.15 \times 20000 + .2 \times 16000 = \$6200$

(b) $f(x) = \begin{cases} .15x & 0 \le x \le 20000 \\ 3000 + .2(x - 20000) & x > 20000 \end{cases}$

(d) $3000 + .2(x - 20000) = 5000$
$.2(x - 20000) = 2000$
$x - 20000 = 10000$
$x = \$30{,}000$

(c)



3. A paperback sells for $12. The author is paid royalties of 10% on the first 10,000 copies sold, and 15% on any additional copies.
   a. When the 6,000th book is sold, how much will the author earned ~~on that sale?~~

$1.2 \times 6000 = \$7200$

   b. When the 12,000th book is sold, how much will the author earned ~~on that sale?~~

$12000 + 1.8(12000 - 10000) = \$15{,}600$

   c. Let x be the number of copies sold. Write a piecewise function for R (the royalty ~~payment earned on that sale~~) in terms of x.

total money earned

$R(x) = \begin{cases} 1.2x & 0 \le x \le 10000 \\ 12000 + 1.8(x - 10000) & x > 10000 \end{cases}$

   d. How many copies have to be sold in order for the author to have earned $30,000?

$12000 + 1.8(x - 10000) = 30000$
$1.8(x - 10000) = 18000$
$x - 10000 = 10000$
$x = 20{,}000$ books

Piecewise Functions *Name:*_____

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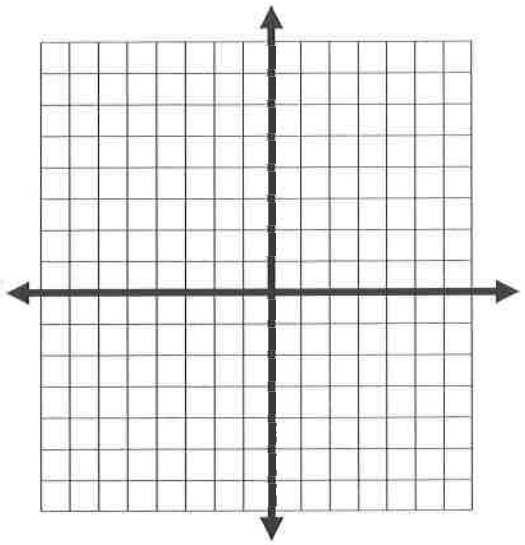


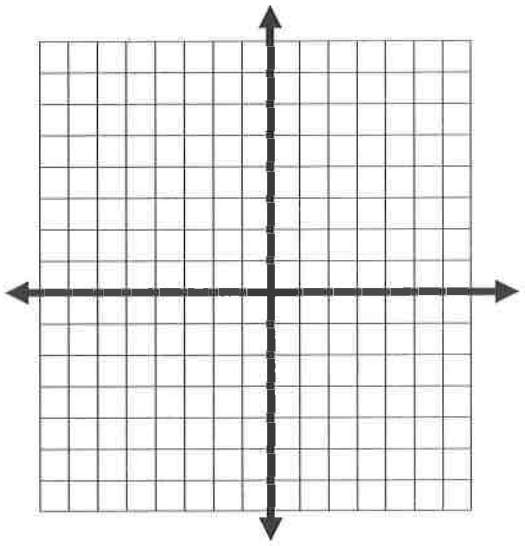
Function? Yes or No

$f(3) =$

$f(-4) =$

$f(-2) =$

2. $f(x) = \begin{cases} 2x+1 & x \geq 1 \\ x^2 + 3 & x < 1 \end{cases}$



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$f(-2) =$

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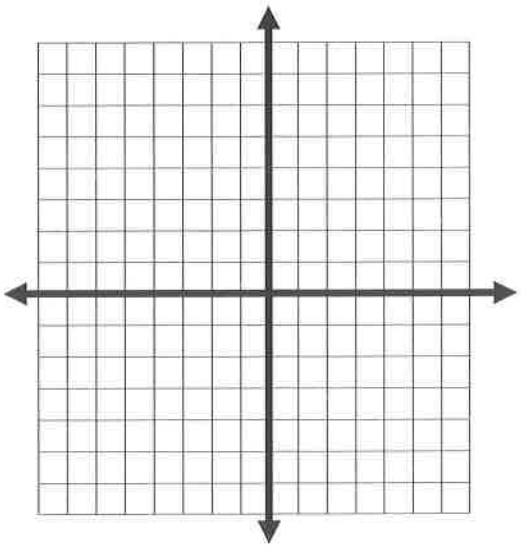

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$f(-4) =$

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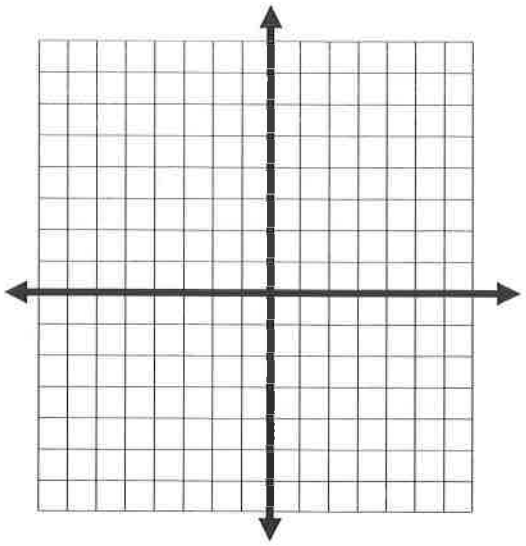

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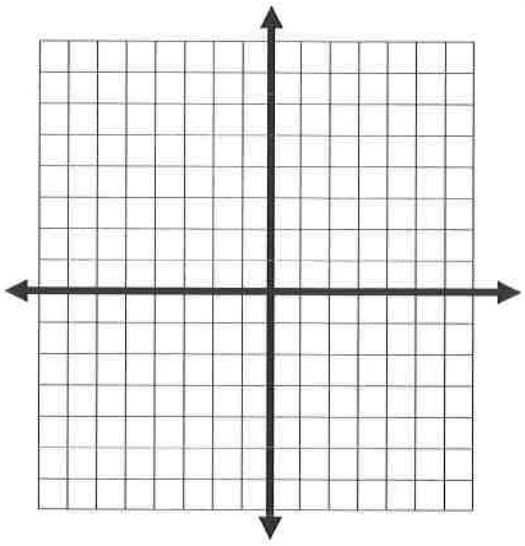

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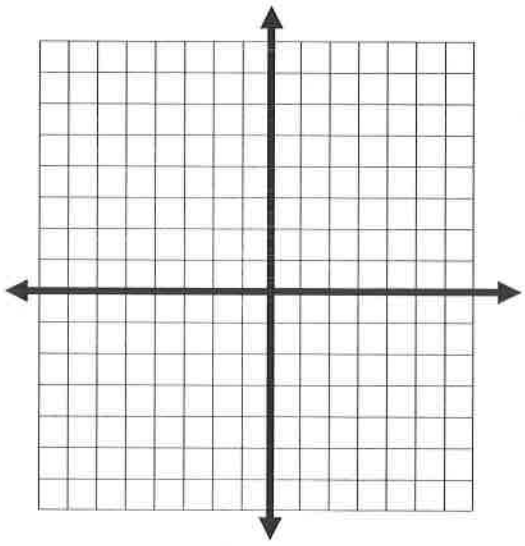

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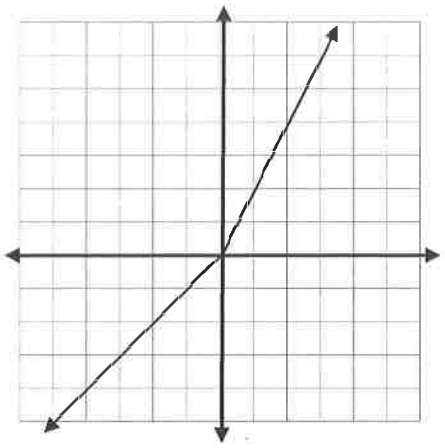

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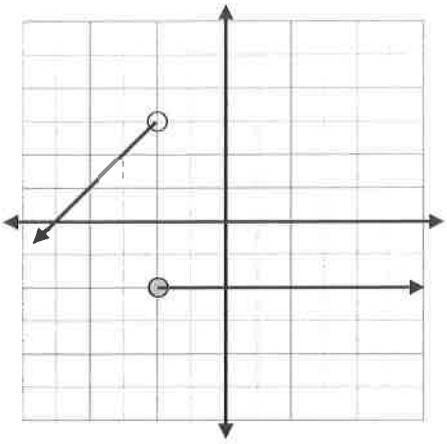

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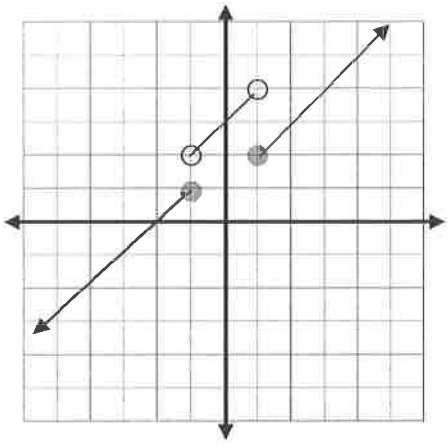

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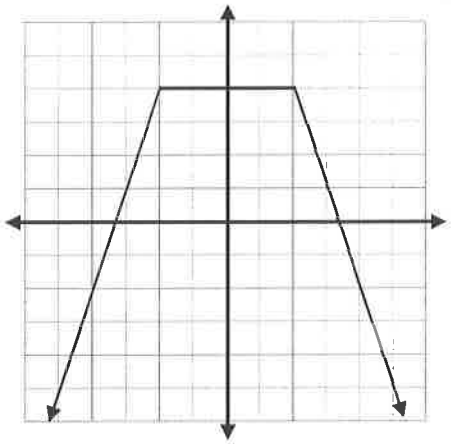

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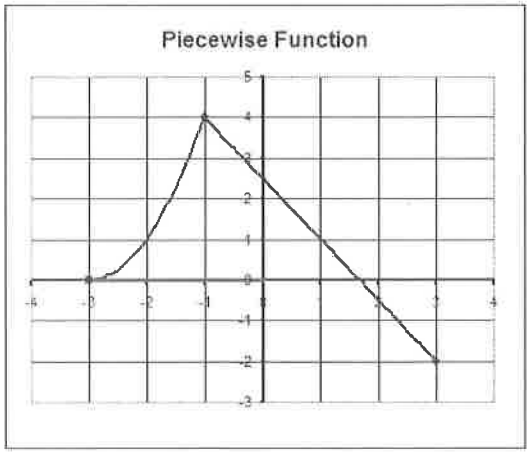

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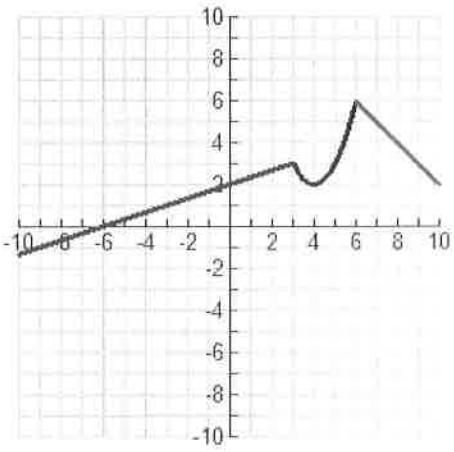

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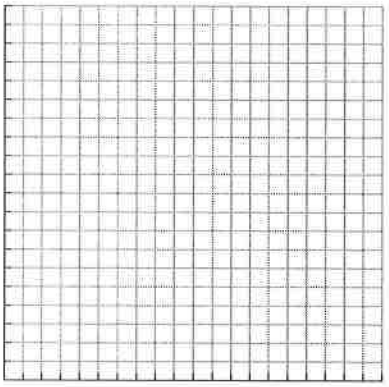

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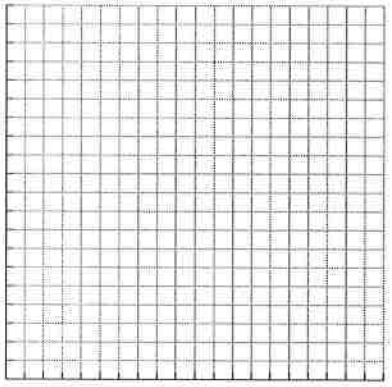

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   b. When the $12{,}000^{th}$ book is sold, how much will the author have earned in total?

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   d. How many copies have to be sold in order for the author to have earned $30,000?