Limits Practice　　　　　　　　　　Name: Key

Find each limit. If the limit does not exist, explain why.

Remember: Take an analytical approach first. Attempt substitution and if this yields a value in indeterminate form, attempt to simplify (factor, expand, use trig relationships, rationalize, find a common denominator, etc…). If indeterminate form still exists after attempts to simplify, analyze the limit graphically. Be sure to include a sketch of your graph when graphing is necessary.

1. $\lim\limits_{x\to 2}\left(8-3x+12x^2\right) = 8-3(2)+12(2)^2 = \boxed{50}$

2. $\lim\limits_{t\to -3}\dfrac{6+4t}{t^2+1} = \dfrac{6+4(-3)}{(-3)^2+1} = \dfrac{-6}{10} = \boxed{-3/5}$

3. $\lim\limits_{x\to -5}\dfrac{x^2-25}{x^2+2x-15} \quad \dfrac{(x+5)(x-5)}{(x+5)(x-3)} = \dfrac{x-5}{x-3} = \dfrac{-10}{-8} = \boxed{5/4}$

4. $\lim\limits_{z\to 8}\dfrac{2z^2-17z+8}{8-z} \quad \dfrac{(2z-1)\cancel{(z-8)}}{-(z-8)} = -(2z-1) = \boxed{-15}$

5. $\lim\limits_{y\to 7}\dfrac{y^2-4y-21}{3y^2-17y-28} \quad \dfrac{(y-7)(y+3)}{(y-7)(3y+4)} = \dfrac{y+3}{3y+4} = \dfrac{10}{25} = \boxed{2/5}$

6. $\lim\limits_{h\to 0}\dfrac{(6+h)^2-36}{h} = \dfrac{36+12h+h^2-36}{h} = \dfrac{h(12+h)}{h} = \boxed{12}$

7. $\lim\limits_{z\to 4}\dfrac{(\sqrt{z}-2)(\sqrt{z}+2)}{(z-4)(\sqrt{z}+2)} = \dfrac{(z-4)}{(z-4)(\sqrt{z}+2)} = \boxed{\dfrac{1}{4}}$

8. $\lim\limits_{x\to -3}\dfrac{(\sqrt{2x+22}-4)(\sqrt{2x+22}+4)}{(x+3)(\sqrt{2x+22}+4)} = \dfrac{2x+22-16}{(x+3)(\sqrt{2x+22}+4)} = \dfrac{2(x+3)}{(x+3)(\sqrt{2x+22}+4)} = \dfrac{2}{8} = \boxed{\dfrac{1}{4}}$

9. $\lim\limits_{x\to 0}\dfrac{x}{(3-\sqrt{x+9})}\cdot\dfrac{(3+\sqrt{x+9})}{(3+\sqrt{x+9})} = \dfrac{x(3+\sqrt{x+9})}{9-(x+9)} = -(3+\sqrt{x+9}) = \boxed{\cancel{6}}\ \boxed{-6}$

1. $\lim\limits_{x\to 0}\dfrac{\tan x}{x} = \dfrac{\sin x}{\cos x}\cdot\dfrac{1}{x} = \dfrac{\sin x}{x}\cdot\dfrac{1}{\cos x} = 1\cdot 1 = \boxed{1}$

2. $\lim\limits_{x\to 0}\dfrac{\sin 4x}{x}\cdot\dfrac{4}{4} = \dfrac{4\sin 4x}{4x} = 4\cdot 1 = \boxed{4}$

3. $\lim\limits_{x\to 0}\dfrac{\sin 4x}{5x}\cdot\dfrac{4/5}{4/5} = \dfrac{\frac{4}{5}\sin 4x}{4x} = \dfrac{4}{5}\cdot 1 = \boxed{\dfrac{4}{5}}$

4. $\lim\limits_{x\to 0}\dfrac{\sin x}{3x}\cdot\dfrac{1/3}{1/3} = \dfrac{\frac{1}{3}\sin x}{x} = \dfrac{1}{3}\cdot 1 = \boxed{1/3}$

5. $\lim\limits_{x\to 0}\dfrac{\tan 2x}{3x} \quad \dfrac{\sin 2x}{3x\cos 2x}\cdot\dfrac{2/3}{2/3} \quad \dfrac{\frac{2}{3}\sin 2x}{2x}\cdot\dfrac{1}{\cos(2x)} = \boxed{\dfrac{2}{3}}$

6. $\lim\limits_{x\to 0}\dfrac{\tan 5x}{\tan 6x} \quad \dfrac{\sin 5x}{\cos 5x}\cdot\dfrac{\cos 6x}{\sin 6x}$ = see back

7. $\lim\limits_{x\to 0}\dfrac{-\cos 4x+1}{x}$ = see back

8. $\lim\limits_{x\to 0}\dfrac{\sec x-1}{x\sec x}$ = see back

9. $\lim\limits_{t\to 0}\dfrac{\sin^2 t}{t^2}$ = see back

10 . Use the Squeeze Theorem to determine the value of $\lim\limits_{x\to 0}x^4\sin\left(\dfrac{\pi}{x}\right) = 0$

$-1 \le \sin\frac{\pi}{x} \le 1$

$-x^4 \le x^4\sin\frac{\pi}{x} \le x^4$

$\lim\limits_{x\to 0} -x^4 = 0 \quad \lim\limits_{x\to 0} x^4 = 0$

10. Given the function

$$f(x)=\begin{cases}7-4x & x<1\\ x^2+2 & x\ge 1\end{cases}$$

Evaluate the following limits, if they exist.

(a) $\lim\limits_{x\to -6}f(x) = 7-4(-6) = \boxed{31}$

(b) $\lim\limits_{x\to 1}f(x)$ $\quad \lim\limits_{x\to 1^-} 7-4(1)=3$

$\lim\limits_{x\to 1^+} (1)^2+2=3$

$\therefore \lim\limits_{x\to 1} f(x)=3$

#6 $\frac{5x \cdot \sin 5x}{5x \cdot \cos 5x} \cdot \frac{\cos 6x \cdot 6x}{\sin 6x \cdot 6x}$

$5x \boxed{\frac{\sin 5x}{5x}} \cdot \frac{1}{\cos 5x} \cdot \boxed{\frac{6x}{6x \sin 6x}} \cdot \frac{\cos 6x}{1}$

$\frac{5\cancel{x}}{\cos 5x} \cdot \frac{\cos 6x}{6\cancel{x}} = \frac{5}{1} \cdot \frac{1}{6} = 5/6$

#7 $\lim_{x \to 0} \frac{(-\cos 4x + 1)}{x} \frac{(-\cos 4x - 1)}{(-\cos 4x - 1)}$

$\lim_{x \to 0} \frac{\cos^2 4x - 1}{x(-\cos 4x - 1)} = \frac{(1 - \sin^2 4x) - 1}{x(-\cos 4x - 1)}$

$\lim_{x \to 0} \frac{-\sin^2 4x}{x(-\cos 4x - 1)} = \frac{-4 \sin^2 4x}{4x(-\cos 4x - 1)}$

$\lim_{x \to 0} \frac{-4 \sin 4x}{-\cos 4x - 1} = \frac{-4(0)}{-1-1} = \frac{0}{-2} = 0$

#8. $\frac{\frac{1}{\cos x} - 1}{\frac{x}{\cos x}} = \frac{\frac{1 - \cos x}{\cos x}}{\frac{x}{\cos x}} = \frac{1 - \cos x}{x} = 0$

#9 $\frac{\sin^2 t}{t^2} = \frac{\sin t}{t} \frac{\sin t}{t} = 1 \cdot 1 = 1$

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2. $\lim\limits_{t\to -3}\dfrac{6+4t}{t^2+1}$

3. $\lim\limits_{x\to -5}\dfrac{x^2-25}{x^2+2x-15}$

4. $\lim\limits_{z\to 8}\dfrac{2z^2-17z+8}{8-z}$

5. $\lim\limits_{y\to 7}\dfrac{y^2-4y-21}{3y^2-17y-28}$

6. $\lim\limits_{h\to 0}\dfrac{(6+h)^2-36}{h}$

7. $\lim\limits_{z\to 4}\dfrac{\sqrt{z}-2}{z-4}$

8. $\lim\limits_{x\to -3}\dfrac{\sqrt{2x+22}-4}{x+3}$

9. $\lim\limits_{x\to 0}\dfrac{x}{3-\sqrt{x+9}}$

10. Given the function

$$f(x)=\begin{cases}7-4x & x<1\\ x^2+2 & x\geq 1\end{cases}$$

Evaluate the following limits, if they exist.

(a) $\lim\limits_{x\to -6} f(x)$ (b) $\lim\limits_{x\to 1} f(x)$

1. $\lim\limits_{x\to 0}\dfrac{\tan x}{x}$

2. $\lim\limits_{x\to 0}\dfrac{\sin 4x}{x}$

3. $\lim\limits_{x\to 0}\dfrac{\sin 4x}{5x}$

4. $\lim\limits_{x\to 0}\dfrac{\sin x}{3x}$

5. $\lim\limits_{x\to 0}\dfrac{\tan 2x}{3x}$

6. $\lim\limits_{x\to 0}\dfrac{\tan 5x}{\tan 6x}$

7. $\lim\limits_{x\to 0}\dfrac{-\cos 4x+1}{x}$

8. $\lim\limits_{x\to 0}\dfrac{\sec x-1}{x\sec x}$

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