

# Limits Assignment

Name: Key

Find each limit. If the limit does not exist, explain why.

Remember: Take an analytical approach first. Attempt substitution and if this yields a value in indeterminate form, attempt to simplify (factor, expand, use trig relationships, rationalize, find a common denominator, etc...). If indeterminate form still exists after attempts to simplify, analyze the limit graphically. Be sure to include a sketch of your graph when graphing is necessary.

<p>1. <math>\lim_{t \rightarrow -5} \frac{t^2 + 6t + 5}{t^2 + 2t - 15} = \frac{0}{0}</math></p> <p>Factor.</p> $\frac{(t+5)(t+1)}{(t+5)(t-3)} = \frac{t+1}{t-3}$ $\lim_{t \rightarrow -5} \frac{t+1}{t-3} = \frac{-4}{-8} = \boxed{\frac{1}{2}}$	<p>2. <math>\lim_{h \rightarrow 0} \frac{(2+h)^3 - 8}{h} = \frac{0}{0}</math></p> <p>expand</p> $\frac{(2+h)(4+4h+h^2) - 8}{h}$ $= \frac{8+4h+8h+4h^2+2h^2+h^3-8}{h}$ $= \frac{h^3+6h^2+12h}{h}$ $\lim_{h \rightarrow 0} h^2+6h+12 = \boxed{12}$	<p>3. <math>\lim_{t \rightarrow 3} \frac{3-t}{\sqrt{t+1} - \sqrt{5t-11}} = \frac{0}{0}</math></p> <p>rationalize</p> $\frac{(3-t)(\sqrt{t+1} + \sqrt{5t-11})}{(t+1) - (5t-11)}$ $= \frac{(3-t)(\sqrt{t+1} + \sqrt{5t-11})}{-4t+12}$ $= \frac{-(t-3)(\sqrt{t+1} + \sqrt{5t-11})}{-4(t-3)}$ $\lim_{t \rightarrow 3} \frac{\sqrt{t+1} + \sqrt{5t-11}}{4} = \frac{4}{4} = \boxed{1}$
<p>4. <math>\lim_{y \rightarrow -1} \frac{\frac{4}{y} \cdot \frac{1}{4+3y} + \frac{1}{y} \cdot \frac{(4+3y)}{(4+3y)}}{y+1} = \frac{0}{0}</math></p> <p>simplify:</p> $\frac{\frac{y+4+3y}{y(4+3y)}}{y+1} = \frac{\frac{4y+4}{y(4+3y)}}{y+1}$ $\frac{4(y+1)}{y(4+3y)} \cdot \frac{1}{(y+1)} = \frac{4}{y(4+3y)}$ $\lim_{y \rightarrow -1} \frac{4}{y(4+3y)} = \boxed{-4}$	<p>5. <math>g(t) = \begin{cases} t^2 - t^3 &amp; t &lt; 2 \\ 5t - 14 &amp; t \geq 2 \end{cases}</math></p> <p><math>\lim_{t \rightarrow 2} g(t)</math></p> $\lim_{t \rightarrow 2^-} 2^2 - 2^3 = -4$ $\lim_{t \rightarrow 2^+} 5(2) - 14 = -4$ $\therefore \lim_{t \rightarrow 2} g(t) = \boxed{-4}$	<p>6. <math>\lim_{z \rightarrow 4} \frac{6z}{2+3z^2}</math></p> <p>Substitute</p> $\lim_{z \rightarrow 4} \frac{6(4)}{2+3(4)^2} = \frac{24}{50} = \boxed{\frac{12}{25}}$

$$7. \lim_{h \rightarrow 0} \frac{\sin(2h)(1 - \cos h)}{h}$$

special limit

$$\sin(2h) \cdot \frac{(1 - \cos h)}{h}$$

$$\lim_{h \rightarrow 0} \sin(2h) \cdot \lim_{h \rightarrow 0} \frac{1 - \cos h}{h}$$

$$= 0 \cdot 0 = \boxed{0}$$

$$8. \lim_{x \rightarrow 0} \frac{\sin(\sin x)}{\sin x}$$

just like  $\frac{\sin x}{x}$  only the  
"x" is " $\sin x$ "

$$\lim_{x \rightarrow 0} = \boxed{1}$$

$$9. \lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x \cos x}$$

rewrite

$$\frac{\frac{\sin x}{\cos x} - \sin x}{x \cos x} = \frac{\frac{\sin x - \sin x \cos x}{\cos x}}{x \cos x}$$

$$= \frac{\sin x (1 - \cos x)}{\cos x} \cdot \frac{1}{x \cos x} = \frac{\sin x}{x} \cdot \frac{(1 - \cos x)}{\cos^2 x}$$

$$= 1 \cdot \frac{0}{1} = \boxed{0}$$

$$10. \lim_{w \rightarrow 8} \frac{w+2}{w^2-6w-16} = \frac{10}{0}$$

Substitute

means an asymptote

$$\lim_{w \rightarrow 8^-} \frac{w+2}{w^2-6w-16} = \frac{-18}{104} = -\infty$$

$$\lim_{w \rightarrow 8^+} \frac{w+2}{w^2-6w-16} = 1 = \infty$$

$$\lim_{w \rightarrow 8} = \text{DNE}$$

$$11. \lim_{w \rightarrow -4} \frac{w^2-16}{(w-2)(w+3)-6} = \frac{0}{0}$$

Factor & Foil

$$\frac{(w-4)(w+4)}{w^2+w-12} = \frac{(w-4)(w+4)}{w^2+w-12}$$

$$= \frac{(w-4)(w+4)}{(w+4)(w-3)} = \frac{w-4}{w-3}$$

$$\lim_{w \rightarrow -4} \frac{w-4}{w-3} = \frac{-8}{-7} = \boxed{\frac{8}{7}}$$

$$12. \lim_{x \rightarrow 0} \frac{\sin^2 x \cos x}{1 - \cos x} = \frac{0}{0}$$

multiply by conjugate

$$\frac{(\sin^2 x \cos x)(1 + \cos x)}{(1 - \cos x)(1 + \cos x)} =$$

$$\frac{(\sin^2 x \cos x)(1 + \cos x)}{1 - \cos^2 x} =$$

$$\frac{\sin^2 x \cos x (1 + \cos x)}{\sin^2 x} =$$

$$\cos x (1 + \cos x)$$

$$\lim_{x \rightarrow 0} = 1(1+1) = \boxed{2}$$

$$13. \lim_{x \rightarrow 2} \frac{x-2}{\sqrt{2}-\sqrt{x}}$$

rationalize

$$\frac{(x-2)(\sqrt{2}+\sqrt{x})}{(\sqrt{2}-\sqrt{x})(\sqrt{2}+\sqrt{x})} =$$

$$\frac{(x-2)(\sqrt{2}+\sqrt{x})}{(2-x)} = \frac{-(2-x)(\sqrt{2}+\sqrt{x})}{(2-x)}$$

$$= -(\sqrt{2}+\sqrt{x})$$

$$\lim_{x \rightarrow 2} = \boxed{-2\sqrt{2}}$$

$$14. \lim_{x \rightarrow 1} (x-1)^2 \cos\left(\frac{1}{x-1}\right)$$

doesn't matter  
what this is

$$\lim_{x \rightarrow 1} (x-1)^2 \cdot \lim_{x \rightarrow 1} \cos\left(\frac{1}{x-1}\right)$$

b/c  $\uparrow$  is 0, the whole  
limit is 0.

$$\lim_{x \rightarrow 1} = \boxed{0}$$

$$15. \lim_{t \rightarrow 0} \frac{t^2+6}{t^2-3}$$

$$\frac{(0)^2+6}{(0)^2-3} = \frac{6}{-3} = \boxed{-2}$$

# Limits Assignment

Name: \_\_\_\_\_

Find each limit. If the limit does not exist, explain why.

Remember: Take an analytical approach first. Attempt substitution and if this yields a value in indeterminate form, attempt to simplify (factor, expand, use trig relationships, rationalize, find a common denominator, etc...). If indeterminate form still exists after attempts to simplify, analyze the limit graphically. Be sure to include a sketch of your graph when graphing is necessary.

1. $\lim_{t \rightarrow -5} \frac{t^2 + 6t + 5}{t^2 + 2t - 15}$	2. $\lim_{h \rightarrow 0} \frac{(2+h)^3 - 8}{h}$	3. $\lim_{t \rightarrow 3} \frac{3-t}{\sqrt{t+1} - \sqrt{5t-11}}$
4. $\lim_{y \rightarrow -1} \frac{\frac{1}{4+3y} + \frac{1}{y}}{y+1}$	5. $g(t) = \begin{cases} t^2 - t^3 & t < 2 \\ 5t - 14 & t \geq 2 \end{cases}$ $\lim_{t \rightarrow 2} g(t)$	6. $\lim_{z \rightarrow 4} \frac{6z}{2+3z^2}$

$$7. \lim_{h \rightarrow 0} \frac{\sin(2h)(1 - \cos h)}{h}$$

$$8. \lim_{x \rightarrow 0} \frac{\sin(\sin x)}{\sin x}$$

$$9. \lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x \cos x}$$

$$10. \lim_{w \rightarrow 8} \frac{w+2}{w^2-6w-16}$$

$$11. \lim_{w \rightarrow -4} \frac{w^2-16}{(w-2)(w+3)-6}$$

$$12. \lim_{x \rightarrow 0} \frac{\sin^2 x \cos x}{1 - \cos x}$$

$$13. \lim_{x \rightarrow 2} \frac{x-2}{\sqrt{2}-\sqrt{x}}$$

$$14. \lim_{x \rightarrow 1} (x-1)^2 \cos\left(\frac{1}{x-1}\right)$$

$$15. \lim_{t \rightarrow 0} \frac{t^2+6}{t^2-3}$$