

Solving trig equations

① $4 \sin(3t) = 2$ $\sin(3t) = \frac{1}{2}$ $3t = \frac{\pi}{6} + 2\pi n$ $t = \frac{\pi}{18} + \frac{2\pi n}{3}$
 $3t = \frac{5\pi}{6} + 2\pi n$ $t = \frac{5\pi}{18} + \frac{2\pi n}{3}$

② $4 \sin(3t) = 2$ in $[0, \frac{4\pi}{3}]$ $n = -1 \ 0 \ 1 \ 2$
 Change to $t = \frac{\pi}{18} + \frac{12\pi n}{18}$ $\frac{\pi}{18}$ $\frac{13\pi}{18}$ $\frac{25\pi}{18}$
 $t = \frac{5\pi}{18} + \frac{12\pi n}{18}$ $\frac{5\pi}{18}$ $\frac{17\pi}{18}$ $\frac{29\pi}{18}$

③ $2 \cos(\frac{x}{3}) + \sqrt{2} = 0$ $\cos(\frac{x}{3}) = -\frac{\sqrt{2}}{2}$ $\frac{x}{3} = \frac{3\pi}{4} + 2\pi n$ $x = \frac{9\pi}{4} + 6\pi n$
 $\frac{x}{3} = \frac{5\pi}{4} + 2\pi n$ $x = \frac{15\pi}{4} + 6\pi n$

④ $2 \cos(\frac{x}{3}) + \sqrt{2} = 0$ in $[-7\pi, 7\pi]$
 change to $x = \frac{9\pi}{4} + \frac{24\pi n}{4}$ $n = -1 \ 0 \ 1$
 $x = \frac{15\pi}{4} + \frac{24\pi n}{4}$ $-\frac{15\pi}{4}$ $\frac{9\pi}{4}$ $\frac{33\pi}{4}$
 $x = -\frac{15\pi}{4}, \frac{9\pi}{4}, \frac{33\pi}{4}$

⑤ $4 \cos(6z) = \sqrt{12}$ in $[0, \frac{\pi}{2}]$ $n = 0 \ 1$
 $4 \cos(6z) = 2\sqrt{3}$ $6z = \frac{\pi}{6} + 2\pi n$ $z = \frac{\pi}{36} + \frac{2\pi n}{36}$ $\frac{\pi}{36}$ $\frac{13\pi}{36}$
 $\cos(6z) = \frac{\sqrt{3}}{2}$ $6z = \frac{11\pi}{6} + 2\pi n$ $z = \frac{11\pi}{36} + \frac{2\pi n}{36}$ $\frac{11\pi}{36}$ $\frac{23\pi}{36}$

⑥ $2 \sin(\frac{3y}{2}) + \sqrt{3} = 0$ in $[-\frac{7\pi}{3}, 0]$ $n = -2 \ -1 \ 0$
 $\sin(\frac{3y}{2}) = -\frac{\sqrt{3}}{2}$ $\frac{3y}{2} = \frac{4\pi}{3} + 2\pi n$ $y = \frac{8\pi}{9} + \frac{4\pi n}{3}$ $-\frac{16\pi}{9}$ $-\frac{4\pi}{9}$ $\frac{8\pi}{9}$
 $\frac{3y}{2} = \frac{5\pi}{3} + 2\pi n$ $y = \frac{10\pi}{9} + \frac{4\pi n}{3}$ $-\frac{14\pi}{9}$ $-\frac{2\pi}{9}$ $\frac{10\pi}{9}$

⑦ $8 + \tan(2x) - 5 = 3$ in $[-\frac{\pi}{2}, \frac{3\pi}{2}]$ $n = -1 \ 0 \ 1$
 $\tan(2x) = 1$ $2x = \frac{\pi}{4} + 2\pi n$ $x = \frac{\pi}{8} + \pi n$ $\frac{\pi}{8}$ $\frac{9\pi}{8}$
 $2x = \frac{5\pi}{4} + 2\pi n$ $x = \frac{5\pi}{8} + \pi n$ $-\frac{3\pi}{8}$ $\frac{5\pi}{8}$ $\frac{13\pi}{8}$

⑧ $16 = -9 \sin(7x) - 4$ in $[-2\pi, \frac{9\pi}{4}]$
 $\sin(7x) = -\frac{20}{9}$ **no solution** $-1 < \sin(x) < 1$

⑨ $\sqrt{3} \tan(\frac{t}{4}) + 5 = 4$ in $[0, 4\pi]$ $n = -1 \ 0$
 $\tan(\frac{t}{4}) = -\frac{1}{\sqrt{3}}$ $\frac{t}{4} = \frac{5\pi}{6} + 2\pi n$ $t = \frac{20\pi}{6} + \frac{48\pi n}{6}$ $\frac{20\pi}{6}$
 $\frac{t}{4} = \frac{11\pi}{6} + 2\pi n$ $t = \frac{44\pi}{6} + \frac{48\pi n}{6}$ $\frac{44\pi}{6}$

⑩ $\sqrt{3} \csc(9z) - 7 = -5$ in $[-\frac{\pi}{3}, \frac{4\pi}{9}]$ $n = -1 \ 0 \ 1$
 $\csc(9z) = \frac{2}{\sqrt{3}}$ $9z = \frac{\pi}{3} + 2\pi n$ $z = \frac{\pi}{27} + \frac{2\pi n}{27}$ $\frac{\pi}{27}$ $\frac{7\pi}{27}$
 $\sin(9z) = \frac{\sqrt{3}}{2}$ $9z = \frac{2\pi}{3} + 2\pi n$ $z = \frac{2\pi}{27} + \frac{2\pi n}{27}$ $\frac{2\pi}{27}$ $\frac{8\pi}{27}$

(11) $1 - 14 \cos\left(\frac{2x}{5}\right) = -6$ in $[5\pi, 40\pi/3] \rightarrow [30\pi/6, 80\pi/6]$ $n = 1, 2$
 $\cos\left(\frac{2x}{5}\right) = \frac{1}{2}$ $\frac{2x}{5} = \pi/3 + 2\pi n$ $x = \frac{5\pi}{6} + \frac{30\pi n}{6}$ $\frac{35\pi}{6}, \frac{65\pi}{6}$
 $\frac{2x}{5} = \frac{5\pi}{3} + 2\pi n$ $x = \frac{25\pi}{6} + \frac{30\pi n}{6}$ $\frac{55\pi}{6}, \frac{85\pi}{6}$
 $n = \frac{35\pi}{6}, \frac{55\pi}{6}, \frac{65\pi}{6}, \frac{85\pi}{6}$

(12) $15 = 17 + 4 \cos\left(\frac{y}{7}\right)$ in $[10\pi, 15\pi] \rightarrow [30\pi/3, 45\pi/3]$ $n = 1$
 $\cos\left(\frac{y}{7}\right) = -\frac{1}{2}$ $\frac{y}{7} = 2\pi/3 + 2\pi n$ $y = \frac{14\pi}{3} + \frac{42\pi n}{3}$ $14\pi/3$
 $\frac{y}{7} = \frac{4\pi}{3} + 2\pi n$ $y = \frac{28\pi}{3} + \frac{42\pi n}{3}$ $28\pi/3$

no solutions in the given interval