Final Limits Practice

Key

Find the following limits. If the limit does not exist, briefly explain why.

1. $\lim_{x\to 2}(x+3)^3 = 125$

2. $\lim_{x\to 1}\frac{x^2-3x+2}{x^2-1}$ $\frac{(x-2)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}} = \frac{-1}{2}$

3. $\lim_{x\to 1}\frac{x^3-1}{x-1}$ $\frac{(x-1)(x^2+x+1)}{(x-1)} = 3$

4. $\lim_{x\to 3^+}\frac{|x-3|}{x-3}$; $\lim_{x\to 3^-}\frac{|x-3|}{x-3}$; $\lim_{x\to 3}\frac{|x-3|}{x-3}$

   $\frac{1}{1}=1$ $\frac{1}{-1}=-1$ DNE

5. $\lim_{x\to\pi} x\cos x$ $-\pi$

6. $\lim_{x\to e^2} 3\ln x$ $3\cdot 2=6$

7. $\lim_{x\to 2^+}\sqrt{x^2-4} = 0$

8. $\lim_{x\to 2^-}\sqrt{x^2-4} = 0$

9. $\lim_{t\to\frac{\pi}{2}}\frac{\cot t}{\csc t}$ $\frac{\frac{\cos t}{\sin t}}{\frac{1}{\sin t}} = \cos t = 0$

10. $\lim_{h\to 0}\left(\frac{(x+h)^2-x^2}{h}\right) = \frac{0}{0}$

    $\frac{x^2+2hx+h^2-x^2}{h} = \frac{h(2x+h)}{h} = 2x+h = \cancel{0}$ $2x$

11. $\lim_{x\to 2}\frac{(\sqrt{2x+5}-3)(\sqrt{2x+5}+3)}{(x-2)(\sqrt{2x+5}+3)} = \frac{2x+5-9}{(x-2)(\sqrt{2x+5}+3)} = \frac{2\cancel{(x-2)}}{\cancel{(x-2)}(\sqrt{2x+5}+3)}$

    $= \frac{2}{6} = \frac{1}{3}$

12. $\lim_{x\to 1^+}\frac{(x-1)(\sqrt{x^2-1})}{\sqrt{x^2-1}(\sqrt{x^2-1})} = \frac{(x-1)(\sqrt{x^2-1})}{(x^2-1)} = \frac{\cancel{(x-1)}(\sqrt{x^2-1})}{\cancel{(x-1)}(x+1)} = \frac{0}{2} = 0$

13. $\lim_{h\to 0}\frac{(\sqrt{4+h}-2)(\sqrt{4+h}+2)}{(h)(\sqrt{4+h}+2)} = \frac{4+h-4}{h(\sqrt{4+h}+2)} = \frac{1}{\sqrt{4+h}+2} = \frac{1}{4}$

14. $\lim_{t\to 3}\frac{\frac{1}{t}-\frac{1}{3}}{t-3}$ $\frac{\frac{3}{3t}-\frac{t}{3t}}{t-3} = \frac{\frac{-(t-3)}{3t}}{t-3} = \frac{-1}{3t} = \frac{-1}{9}$

15. $\lim_{x\to\infty}\frac{x^2}{3x^2+4}$ $\frac{x^2}{x^2(3+\frac{4}{x^2})} = \frac{1}{3+\frac{4}{x^2}} = \frac{1}{3+0} = \frac{1}{3}$

16. $\lim_{x\to 3^+}\frac{x}{x-3}$  $=\infty$

17. $\lim_{x\to\infty}\frac{3x^2}{1+x^3}$ $\frac{x^3(\frac{3}{x})}{x^3(\frac{1}{x^3}+1)} = \frac{0}{1} = 0$

18. $\lim_{x\to\infty}\frac{1}{|x|} = 0$

19. $\lim_{x\to 2}\frac{x-2}{x^2+5} = \frac{0}{9} = 0$

20. $\lim_{x\to\infty}\sin x$ DNE

21. $\lim\limits_{x\to 0}\dfrac{\left(\sqrt{1+x}-1\right)\left(\sqrt{1+x}+1\right)}{x\left(\sqrt{1+x}+1\right)}=\dfrac{x}{x\left(\sqrt{1+x}+1\right)}=\dfrac{1}{2}$

22. $\lim\limits_{x\to 0}\dfrac{\dfrac{1}{x+2}-\dfrac{1}{2}}{x}$ $\dfrac{\dfrac{2}{2(x+2)}-\dfrac{(x+2)}{2(x+2)}}{x}=\dfrac{\dfrac{-x}{2(x+2)}}{x}=\dfrac{-1}{4}$

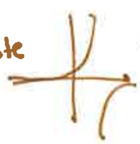
23. $\lim\limits_{x\to 2}\sqrt{\dfrac{x^3-8}{x-2}}$ $\sqrt{\dfrac{(x-2)(x^2+2x+4)}{(x-2)}}=\sqrt{12}=2\sqrt{3}$

24. $\lim\limits_{x\to 1}\dfrac{(x-1)\left(\sqrt{x^2-1}\right)}{\sqrt{x^2-1}\left(\sqrt{x^2-1}\right)}=\dfrac{\cancel{(x-1)}\left(\sqrt{x^2-1}\right)}{\cancel{(x-1)}(x+1)}=\dfrac{0}{2}=0$

25. $\lim\limits_{x\to 0}\dfrac{\left(\sqrt{2+x}-\sqrt{2}\right)\left(\sqrt{2+x}+\sqrt{2}\right)}{x\left(\sqrt{2+x}+\sqrt{2}\right)}=\dfrac{x}{x\left(\sqrt{2+x}+\sqrt{2}\right)}=\dfrac{1}{2\sqrt{2}}=\dfrac{\sqrt{2}}{4}$

26. $\lim\limits_{x\to 2^-}\dfrac{\sqrt{4-x^2}}{\sqrt{6-5x+x^2}}=\sqrt{\dfrac{(2-x)(2+x)}{-(2-x)(x-3)}}=\sqrt{\dfrac{2+x}{-x+3}}=\sqrt{\dfrac{4}{1}}=2$

27. $\lim\limits_{x\to 2}\dfrac{x^2}{4-x^2}$ $\dfrac{x^2}{x^2\left(\frac{4}{x^2}-1\right)}=\dfrac{1}{0}$ asymptote
DNE



28. $\lim\limits_{x\to\infty}\dfrac{x^2+1}{x+1}$ $\dfrac{x\left(x+\frac{1}{x}\right)}{x\left(1+\frac{1}{x}\right)}=\dfrac{\infty+0}{1+0}=\infty$

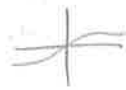
29. $\lim\limits_{x\to 5}\dfrac{(x+3)^{5/3}}{\sqrt{15+2x}}$ $\dfrac{32}{5}$

30. $\lim\limits_{x\to\infty}2^{1/x}=2^0=1$

31. $\lim\limits_{x\to\infty}\left(\sqrt{x^2+x}-x\right)\cdot\dfrac{\left(\sqrt{x^2+x}+x\right)}{\sqrt{x^2+x}+x}=\dfrac{x}{x+\sqrt{x+x^2}}=\dfrac{x}{x+\sqrt{x^2\left(1+\frac{1}{x}\right)}}$
$=\dfrac{x}{|x|\left(\sqrt{1+\frac{1}{x}}+1\right)}=\dfrac{x}{x\left(\sqrt{1+\frac{1}{x}}+1\right)}=\dfrac{1}{\sqrt{1+\frac{1}{x}}+1}=\dfrac{1}{\sqrt{1+0}+1}=\boxed{\dfrac{1}{2}}$
since $x\to\infty$

32. $\lim\limits_{x\to\infty}\left(\sqrt{x^2+1}-x\right)\cdot\dfrac{\left(\sqrt{x^2+1}+x\right)}{\left(\sqrt{x^2+1}+x\right)}=\dfrac{1}{\left(\sqrt{x^2+1}+x\right)}=\dfrac{1}{\sqrt{x^2\left(1+\frac{1}{x^2}\right)}+x}=$
$\dfrac{1}{|x|\sqrt{\left(1+\frac{1}{x^2}\right)}+x}=\dfrac{1}{x\left(\sqrt{1+\frac{1}{x^2}}+1\right)}=\dfrac{1}{\infty\left(\sqrt{1+0}+1\right)}=\dfrac{1}{\infty\cdot 2}=\boxed{0}$

33. $\lim\limits_{x\to\infty}\tan^{-1}x=\dfrac{\pi}{2}$



34. $\lim\limits_{x\to 1}\arccos\left(\dfrac{x}{2}\right)$ $-\cos^{-1}\left(\frac{x}{2}\right)=\cos^{-1}\left(\lim\limits_{x\to 1}\left(\frac{x}{2}\right)\right)=\cos^{-1}\left(\frac{1}{2}\right)=\dfrac{\pi}{3}$

35. $\lim\limits_{x\to 0^+}\ln x=-\infty$

36. $\lim\limits_{x\to\infty}\dfrac{5}{1+3e^{-x}}=\dfrac{5}{1+0}=5$

SQUEEZE THEOREM

37. $\lim\limits_{x\to\infty}\dfrac{\sin x}{x}$ $-\dfrac{1}{x}\le\dfrac{\sin x}{x}\le\dfrac{1}{x}$ $\lim\limits_{x\to\infty}\dfrac{\sin x}{x}=0$

38. $\lim\limits_{x\to 0}\dfrac{\sin x}{x}=1$

39. $\lim\limits_{x\to 0}\dfrac{\sin 3x}{\sin 4x}$ $\dfrac{3x\sin 3x}{3x}\cdot\dfrac{4x}{4x\sin 4x}=\dfrac{3}{4}$

40. $\lim\limits_{x\to\infty}\left(1+\dfrac{1}{x}\right)^x=e$

41. $\lim_{x\to 0}\frac{\sin 4x}{\tan 2x}$ $\frac{\sin 4x}{\frac{\sin 2x}{\cos 2x}} = \frac{4x \sin 4x \cos 2x}{2x \cdot 4x \sin 2x} = 2\cos 2x = 2$

44. $\lim_{x\to\infty}\left(1+\frac{2}{3x}\right)^{5x} = e^{\frac{2}{3}\cdot 5} = e^{10/3}$

42. $\lim_{x\to 0}\frac{(1-\cos x)(1+\cos x)}{x\,(1+\cos x)} = \frac{1-\cos^2 x}{x(1+\cos x)} = \frac{\sin^2 x}{x}\cdot\frac{1}{(1+\cos x)}$

$\frac{\sin x}{x}\cdot\frac{\sin x}{1+\cos x} = \frac{0}{2} = 0$

45. $\lim_{x\to 0}(\ln(\cos x)) = \ln(\cos(0)) = \ln(1) = 0$

43. $\lim_{x\to\infty}\cos\left(\frac{1}{x}\right)$ $\cos \lim_{x\to\infty}\left(\frac{1}{x}\right) = \cos 0 = 1$

Take the limit of the argument of cos to plug that answer in

46. $\lim_{x\to\infty}\left(\cos^{-1}\left(\frac{x}{x^3+2}+\frac{x}{2x+1}\right)\right) = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

$\frac{x^3(\frac{1}{x^2})}{x^3(1+\frac{2}{x^3})} + \frac{x \cdot 1}{x(2+\frac{1}{x})} = \frac{0}{1} + \frac{1}{2}$

Find the following limits for the piecewise function: $f(x)=\begin{cases} x+1, & x<2 \\ x^2-2, & 2<x<4 \\ \sqrt{x+5}, & x\ge 4\end{cases}$

47. $\lim_{x\to 1^+} f(x) = 2$ $\quad \lim_{x\to 1^-} f(x) = 2$ $\quad \lim_{x\to 1} f(x) = 2$ $\quad f(1) = 2$

48. $\lim_{x\to 2^+} f(x) = 2$ $\quad \lim_{x\to 2^-} f(x) = 3$ $\quad \lim_{x\to 2} f(x) =$ DNE $\quad f(2) =$ DNE

49. $\lim_{x\to 3^+} f(x) = 7$ $\quad \lim_{x\to 3^-} f(x) = 7$ $\quad \lim_{x\to 3} f(x) = 7$ $\quad f(3) = 7$

50. $\lim_{x\to 4^+} f(x) = 3$ $\quad \lim_{x\to 4^-} f(x) = 14$ $\quad \lim_{x\to 4} f(x) =$ DNE $\quad f(4) = 3$