Review : Functions

For problems 1 – 4 the given functions perform the indicated function evaluations. (d) $3-5(6-t)-2(6-t)^2$
$3-30+5t-2(36-12t+t^2)$
$3-30+5t-72+24t-2t^2$
$-99+29t-2t^2$

1. $f(x)=3-5x-2x^2$

   (a) $f(4) = -49$ (b) $f(0)=3$ (c) $f(-3)=0$

   (d) $f(6-t) = -99+29t-2t^2$ (e) $f(7-4x)$ $-130+132x-32x^2$ (f) $f(x+h)$ $3-5x-5h-2x^2-4xh-2h^2$

2. $g(t)=\dfrac{t}{2t+6}$

   (a) $g(0)=0$ (b) $g(-3)$ DNE (c) $g(10)=5/13$

   (d) $g(x^2)=\dfrac{x^2}{2x^2+6}$ (e) $g(t+h)=\dfrac{t+h}{2t+2h+6}$ (f) $g(t^2-3t+1)=\dfrac{t^2-3t+1}{2t^2-6t+8}$

3. $h(z)=\sqrt{1-z^2}$ $1-(z^4-4z^3+4z^2)$
$1-z^4+4z^3-4z^2$

   (a) $h(0)=1$ (b) $h(-\frac{1}{2})=\dfrac{\sqrt{3}}{2}$ (c) $h(\frac{1}{2})=\dfrac{\sqrt{3}}{2}$

   (d) $h(9z)=\sqrt{1-81z^2}$ (e) $h(z^2-2z)$ $\sqrt{1-4z^2+4z^3-z^4}$ (f) $h(z+k)$ $\sqrt{1-z^2-2zk-k^2}$

4. $R(x)=\sqrt{3+x}-\dfrac{4}{x+1}$

   (a) $R(0)=\sqrt{3}-4$ (b) $R(6)=\dfrac{17}{7}$ (c) $R(-9)$ = no real answer

   (d) $R(x+1)=\sqrt{4+x}-\dfrac{4}{x+2}$ (e) $R(x^4-3)=$ $x^2-\dfrac{4}{x^4-2}$ (f) $R(\frac{1}{x}-1)=\sqrt{2+\frac{1}{x}}-4x$

The **difference quotient** of a function $f(x)$ is defined to be, What is the difference quotient?
not in notes

$$\frac{f(x+h)-f(x)}{h}$$

Slope of the secant line for $(x, f(x))$ and $(x+h, f(x+h))$

For problems 5 – 9 compute the difference quotient of the given function.

5. $f(x)=4x-9$ $\dfrac{4(x+h)-9-(4x-9)}{h}=\dfrac{4x+4h-9-4x+9}{h}=\dfrac{4h}{h}=4$

6. $g(x)=6-x^2$ $\dfrac{6-(x+h)^2-(6-x^2)}{h}=\dfrac{6-(x^2+2xh+h^2)-6+x^2}{h}\;\dfrac{6-x^2-2xh-h^2-6+x^2}{h}=\dfrac{h(-2x-h)}{h}$
$-2x-h$

7. $f(t)=2t^2-3t+9$ $\dfrac{2(t+h)^2-3(t+h)+9-(2t^2-3t+9)}{h}\;\dfrac{2(t^2+2th+h^2)-3t-3h+9-2t^2+3t-9}{h}$
$\dfrac{2t^2+4th+2h^2-3t-3h+9-2t^2+3t-9}{h}=\dfrac{4th+2h^2-3h}{h}=4t+2h-3$

8. $y(z) = \frac{1}{z+2}$

$\frac{1}{z+h+2} - \frac{1}{z+2} = \frac{z+2-(z+h+2)}{(z+h+2)(z+2)} = \frac{-h}{(z+h+2)(z+2)} = \frac{-1}{(z+h+2)(z+2)}$ $\div h$

9. $A(t) = \frac{2t}{3-t}$

$\frac{2(t+h)}{3-(t+h)} - \frac{2t}{3-t} = \frac{(2t+2h)(3-t)-(2t)(3-t-h)}{(3-t-h)(3-t)} = \frac{\cancel{6t}+6h-\cancel{2t^2}-\cancel{2th}-\cancel{6t}+\cancel{2t^2}+\cancel{2th}}{(3-t-h)(3-t)}$ $\div h$

$\frac{6}{(3-t-h)(3-t)}$

For problems 10 – 17 determine all the roots of the given function.

F 10. $f(x) = x^5 - 4x^4 - 32x^3$ $x^3(x^2-4x-32) = x^3(x-8)(x+4) = 0$

roots $x=0$, $x=8$, $x=-4$

F 11. $R(y) = 12y^2 + 11y - 5$ $(4y+5)(3y-1) = 0$

$y = -5/4$ $y = 1/3$

Q 12. $h(t) = 18 - 3t - 2t^2$ $x = -\frac{3}{4}(1 \pm \sqrt{17})$

F,Q 13. $g(x) = x^3 + 7x^2 - x$ $x(x^2+7x-1)$

$x = 0$ $x = \frac{-7 \pm \sqrt{53}}{2}$

F 14. $W(x) = x^4 + 6x^2 - 27$ $(x^2-3)(x^2+9)$

$x^2 = 3$ non-real

$x = \pm\sqrt{3}$

F 15. $f(t) = t^{\frac{5}{3}} - 7t^{\frac{4}{3}} - 8t$ $t(t^{2/3} - 7t^{1/3} - 8) = t(t^{1/3}-8)(t^{1/3}+1)$

$t=0$, $t=512$, $t=-1$

F 16. $h(z) = \frac{z}{z-5} - \frac{4}{z-8}$ $z(z-8) - 4(z-5) = 0$ $(z-10)(z-2) = 0$ check originals for domain issues

$z^2 - 8z - 4z + 20 = 0$ $z=10$, $z=2$

$z^2 - 12z + 20 = 0$

Q 17. $g(w) = \frac{2w}{w+1} + \frac{w-4}{2w-3}$ $2w(2w-3) + (w+1)(w-4) = 0$ $w = \frac{9 \pm \sqrt{161}}{10}$

$4w^2 - 6w + w^2 - 3w - 4 = 0$

$5w^2 - 9w - 4 = 0$

For problems 18 – 22 find the domain and range of the given function.

18. $Y(t) = 3t^2 - 2t + 1$ D: $(-\infty, \infty)$ R: $\left[\frac{2}{3}, \infty\right)$ vertex $= \frac{2}{2(3)} = \frac{2}{6} = \frac{1}{3}$ $Y(\frac{1}{3}) = 3(\frac{1}{3})^2 - 2(\frac{1}{3}) + 1 = \frac{2}{3}$

19. $g(z) = -z^2 - 4z + 7$ D: $(-\infty, \infty)$ R: $(-\infty, 11]$ vertex $= \frac{4}{2(-1)} = -2$ $g(-2) = -(-2)^2 - 4(-2) + 7 = 11$

20. $f(z) = 2 + \sqrt{z^2 + 1}$ D: $(-\infty, \infty)$ R: $[3, \infty)$ smallest $z^2 = 0$ $2 + \sqrt{0+1} = 3$

21. $h(y) = -3\sqrt{14 + 3y}$ D: $[-14/3, \infty)$ R: $(-\infty, 0]$ root can be ≥ 0

$14 + 3y \geq 0$

$3y \geq -14$

$y \geq -14/3$

22. $M(x) = 5 - |x + 8|$ D: $(-\infty, \infty)$ R: $(-\infty, 5]$ $5 - 0 = 5$

5 - larger = # less than 5

For problems 23 – 31 find the domain of the given function.

23. $f(w)=\dfrac{w^3-3w+1}{12w-7}$ D: $\{\mathbb{R} \mid x \neq \frac{7}{12}\}$

F 24. $R(z)=\dfrac{5}{z^3+10z^2+9z}$ $z(z^2+10z+9)$ $z(z+9)(z+1)$ D: $\{\mathbb{R} \mid x \neq 0,-9,-1\}$

Q 25. $g(t)=\dfrac{6t-t^3}{7-t-4t^2}$ D: $\{\mathbb{R} \mid x \neq -\frac{1}{8}(1 \pm \sqrt{113})$

26. $g(x)=\sqrt{25-x^2}$ $25-x^2 \geq 0$ $25 \geq x^2$ D: $-5 \leq x \leq 5$

inside check only

F 27. $h(x)=\sqrt{x^4-x^3-20x^2}$ $x^2(x^2-x-20)$ $x^2(x-5)(x+4)$

$R(-5)=250$ $R(-1)=-18$ $R(1)=-20$ $R(6)=360$

$\boxed{-4}$ $\boxed{0}$ $\boxed{5}$

$+$ $-$ $-$ $+$

D: $(-\infty,-4] \cup [0] \cup [5,\infty)$

making this root zero is ok for domain

F, Q 28. $P(t)=\dfrac{5t+1}{\sqrt{t^3-t^2-8t}}$ $t(t^2-t-8)$ $t=0, t=\frac{1 \pm \sqrt{33}}{2}$

$R(-3)=-12$ $R(-1)=6$ $R(1)=-8$ $R(4)=16$

$\frac{1-\sqrt{33}}{2}$ 0 $\frac{1+\sqrt{33}}{2}$

$-$ $+$ $-$ $+$

D: $\left(\frac{1-\sqrt{33}}{2}, 0\right) \cup \left(\frac{1+\sqrt{33}}{2}, \infty\right)$

29. $f(z)=\sqrt{z-1}+\sqrt{z+6}$ $z \geq 1$ and $z \geq -6$ D: $[1,\infty)$

30. $h(y)=\sqrt{2y+9}-\dfrac{1}{\sqrt{2-y}}$ $2y+9 \geq 0$ $2y \geq -9$ $y \geq -\frac{9}{2}$ $2-y>0$ $2>y$ D: $[-\frac{9}{2}, 2)$

why not test here? AND

31. $A(x)=\dfrac{4}{x-9}-\sqrt{x^2-36}$ $x \neq 9$ $x^2-36 \geq 0$ $x^2 \geq 36$ $x \leq -6, x \geq 6$ D: $(-\infty,-6] \cup [6,\infty), x \neq -9$

32. $Q(y)=\sqrt{y^2+1}-\sqrt[3]{1-y}$ D: $(-\infty,\infty)$

always positive always calculable

For problems 33 – 36 compute $(f \circ g)(x)$ and $(g \circ f)(x)$ for each of the given pair of functions.

33. $f(x)=4x-1$, $g(x)=\sqrt{6+7x}$ $(f \circ g)(x)= 4(\sqrt{6+7x})$ $(g \circ f)(x)=\sqrt{6+7(4x-1}$

34. $f(x)=5x+2$, $g(x)=x^2-14x$ $(f \circ g)(x)= 5(x^2-14x)+$ $(g \circ f)(x)= (5x+2)^2-1$ -24

35. $f(x)=x^2-2x+1$, $g(x)=8-3x^2$ $(f \circ g)(x)= (8-3x^2)^2$ $(g \circ f)(x)= 8-3(x^2$ $+1)$

$8-3(x^2-2x+1)^2$

$(x^2-2x+1)(x^2-2x+1)$

$x^4-2x^3+x^2$

$-2x^3+4x^2-2x$

$+x^2-2x+1$

$-3(x^4-4x^3+6x^2-4x+1)$

$8-3x^4+12x^3-18x^2+12x-3$

$-3x^4+12x^3-18x^2+12x+5$

36. $f(x)=x^2+3,\; g(x)=\sqrt{5+x^2}$

$(f\circ g)(x)=\left(\sqrt{5+x^2}\right)^2+3 = 5+x^2+3 = x^2+8$

$(g\circ f)(x)=\sqrt{5+(x^2+3)^2} = \sqrt{5+x^4+6x^2+9} = \sqrt{x^4+6x^2+14}$

Review : Inverse Functions

For each of the following functions find the inverse of the function. Verify your inverse by computing one or both of the composition as discussed in this section.

Should I write $\frac{x-15}{6}$ as $\frac{1}{6}(x-15)$?

1. $f(x)=6x+15$ $\quad x=6y+15 \quad \frac{x-15}{6}=f'(x)$

2. $h(x)=3-29x$ $\quad x=3-29y \quad h'(x)=\frac{x-3}{-29}$ or $h'(x)=\frac{1}{29}(3-x)$

3. $R(x)=x^3+6$ $\quad x=y^3+6 \quad R'(x)=\sqrt[3]{x-6}$

4. $g(x)=4(x-3)^5+21$ $\quad x=4(y-3)^5+21 \quad g'(x)=\sqrt[5]{\frac{x-21}{4}}+3$

5. $W(x)=\sqrt[5]{9-11x}$ $\quad x=\sqrt[5]{9-11y} \quad W'(x)=-\frac{1}{11}(x^5-9)$ or $W'(x)=\frac{1}{11}(9-x^5)$

6. $f(x)=\sqrt[7]{5x+8}$ $\quad x=\sqrt[7]{5x+8} \quad f'(x)=\frac{1}{5}(x^7-8)$

7. $h(x)=\frac{1+9x}{4-x}$ $\quad x=\frac{1+9y}{4-y} \quad 4x-xy=1+9y \quad 4x-1=9y+xy \quad 4x-1=y(9+x) \quad h'(x)=\frac{4x-1}{9+x}$

8. $f(x)=\frac{6-10x}{8x+7}$ $\quad x=\frac{6-10y}{8y+7} \quad 8xy+7x=6-10y \quad 8xy+10y=6-7x \quad y(8x+10)=6-7x \quad f'(x)=\frac{6-7x}{8x+10}$

Review : Trig Functions

Determine the exact value of each of the following without using a calculator.

Note that the point of these problems is not really to learn how to find the value of trig functions but instead to get you comfortable with the unit circle since that is a very important skill that will be needed in solving trig equations.

1. $\cos\left(\frac{5\pi}{6}\right)$ $\quad -\frac{\sqrt{3}}{2}$

$\frac{1}{2} \div \frac{\sqrt{3}}{2} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

(cos, sin)

$\left(\frac{a}{b}, \frac{c}{b}\right)$

tan $\frac{\frac{c}{b}}{\frac{a}{b}} = \frac{c}{b} \div \frac{a}{b} = \frac{c}{b} \cdot \frac{b}{a} = \frac{c}{a}$

2. $\sin\left(-\frac{4\pi}{3}\right)$ $\quad \frac{\sqrt{3}}{2}$

3. $\sin\left(\frac{7\pi}{4}\right)$ $\quad -\frac{\sqrt{2}}{2}$

4. $\cos\left(-\frac{2\pi}{3}\right)$ $\quad -\frac{1}{2}$

5. $\tan\left(\frac{3\pi}{4}\right)$ $\quad \frac{\sqrt{2}}{2} \cdot \frac{-2}{\sqrt{2}} = -1$

6. $\sec\left(-\frac{11\pi}{6}\right)$ $\quad \cos\left(-\frac{11\pi}{6}\right) \frac{\sqrt{3}}{2}$ $\quad \sec\left(-\frac{11\pi}{6}\right) \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

7. $\cos\left(\frac{8\pi}{3}\right)$ $\quad -\frac{1}{2}$

8. $\tan\left(-\frac{\pi}{3}\right)$ $\quad -\frac{\sqrt{3}}{2} \cdot \frac{2}{1} = -\sqrt{3}$

9. $\tan\left(\frac{15\pi}{4}\right)$ $\quad -\frac{\sqrt{2}}{\sqrt{2}} = -1$

10. $\sin\left(-\frac{11\pi}{3}\right)$ $\quad \frac{\sqrt{3}}{2}$

11. $\sec\left(\frac{29\pi}{4}\right)$ $\quad \cos\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$ $\quad \sec\left(\frac{5\pi}{4}\right) = \frac{-2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{-2\sqrt{2}}{2} = -\sqrt{2}$

Review : Solving Trig Equations

Without using a calculator find the solution(s) to the following equations. If an interval is given then find only those solutions that are in the interval. If no interval is given then find all solutions to the equation.

See extra sheet

1. $4\sin(3t) = 2$

2. $4\sin(3t) = 2$ in $\left[0, \frac{4\pi}{3}\right]$

3. $2\cos\left(\frac{x}{3}\right) + \sqrt{2} = 0$

4. $2\cos\left(\frac{x}{3}\right) + \sqrt{2} = 0$ in $[-7\pi, 7\pi]$

5. $4\cos(6z) = \sqrt{12}$ in $\left[0, \frac{\pi}{2}\right]$

6. $2\sin\left(\frac{3y}{2}\right) + \sqrt{3} = 0$ in $\left[-\frac{7\pi}{3}, 0\right]$

7. $8\tan(2x) - 5 = 3$ in $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$

8. $16 = -9\sin(7x) - 4$ in $\left[-2\pi, \frac{9\pi}{4}\right]$

9. $\sqrt{3}\tan\left(\frac{t}{4}\right) + 5 = 4$ in $[0, 4\pi]$

10. $\sqrt{3}\csc(9z) - 7 = -5$ in $\left[-\frac{\pi}{3}, \frac{4\pi}{9}\right]$

11. $1 - 14\cos\left(\frac{2x}{5}\right) = -6$ in $\left[5\pi, \frac{40\pi}{3}\right]$

12. $15 = 17 + 4\cos\left(\frac{y}{7}\right)$ in $[10\pi, 15\pi]$

Solving trig equations

① $4\sin(3t)=2$ $\quad \sin(3t)=\frac{1}{2}$ $\quad 3t=\frac{\pi}{6}+2\pi n$, $3t=\frac{5\pi}{6}+2\pi n$ $\quad \boxed{t=\frac{\pi}{18}+\frac{2\pi n}{3},\ t=\frac{5\pi}{18}+\frac{2\pi n}{3}}$

② $4\sin(3t)=2$ in $[0,\frac{4\pi}{3}] \to \frac{24\pi}{18}$

change to $t=\frac{\pi}{18}+\frac{12\pi n}{18}$, $t=\frac{5\pi}{18}+\frac{12\pi n}{18}$

n=	-1	0	1	2
	$\cancel{\frac{-11\pi}{18}}$	$\frac{\pi}{18}$	$\frac{13\pi}{18}$	$\cancel{\frac{25\pi}{18}}$
	$\cancel{}$	$\frac{5\pi}{18}$	$\frac{17\pi}{18}$	$\cancel{\frac{29\pi}{18}}$

$\boxed{\frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}}$

③ $2\cos\left(\frac{x}{3}\right)+\sqrt{2}=0$ $\quad \cos\left(\frac{x}{3}\right)=-\frac{\sqrt{2}}{2}$ $\quad \frac{x}{3}=\frac{3\pi}{4}+2\pi n$, $\frac{x}{3}=\frac{5\pi}{4}+2\pi n$ $\quad \boxed{x=\frac{9\pi}{4}+6\pi n,\ x=\frac{15\pi}{4}+6\pi n}$

④ $2\cos\left(\frac{x}{3}\right)+\sqrt{2}=0$ in $[-7\pi, 7\pi]$ $\quad -\frac{28\pi}{4}$ $\quad \frac{28\pi}{4}$

change to $x=\frac{9\pi}{4}+\frac{24\pi n}{4}$, $x=\frac{15\pi}{4}+\frac{24\pi n}{4}$

n=	-1	0	1
	$-\frac{15\pi}{4}$	$\frac{9\pi}{4}$	$\cancel{}$
	$-\frac{9\pi}{4}$	$\frac{15\pi}{4}$	$\cancel{}$

$\boxed{x=-\frac{15\pi}{4}, -\frac{9\pi}{4}, \frac{9\pi}{4}, \frac{15\pi}{4}}$

⑤ $4\cos(6z)=\sqrt{12}$ in $[0,\frac{\pi}{2}] \to \frac{18\pi}{36}$

$4\cos(6z)=2\sqrt{3}$ $\quad 6z=\frac{\pi}{6}+2\pi n$ $\quad z=\frac{\pi}{36}+\frac{12\pi n}{36}$

$\cos(6z)=\frac{\sqrt{3}}{2}$ $\quad 6z=\frac{11\pi}{6}+2\pi n$ $\quad z=\frac{11\pi}{36}+\frac{12\pi n}{36}$

n=	0	1
	$\frac{\pi}{36}$	$\frac{13\pi}{36}$
	$\frac{11\pi}{36}$	$\cancel{\frac{23\pi}{36}}$

$\boxed{z=\frac{\pi}{36}, \frac{11\pi}{36}, \frac{13\pi}{36}}$

⑥ $2\sin\left(\frac{3y}{2}\right)+\sqrt{3}=0$ in $[-\frac{7\pi}{3}, 0]$ $\quad -\frac{21\pi}{9}$ $\quad \frac{12\pi n}{9}$

$\sin\left(\frac{3y}{2}\right)=-\frac{\sqrt{3}}{2}$ $\quad \frac{3y}{2}=\frac{4\pi}{3}+2\pi n$ $\quad y=\frac{8\pi}{9}+\frac{4\pi n}{3}$

$\frac{3y}{2}=\frac{5\pi}{3}+2\pi n$ $\quad y=\frac{10\pi}{9}+\frac{4\pi n}{3}$

n=	-2	-1	0
	$-\frac{16\pi}{9}$	$-\frac{4\pi}{9}$	$\cancel{\frac{8\pi}{9}}$
	$-\frac{14\pi}{9}$	$-\frac{2\pi}{9}$	$\cancel{\frac{10\pi}{9}}$

$\boxed{n=-\frac{16\pi}{9}, -\frac{14\pi}{9}, -\frac{4\pi}{9}, -\frac{2\pi}{9}}$

⑦ $8\tan(2x)-5=3$ in $[-\frac{\pi}{2}, \frac{3\pi}{2}] \to [-\frac{4\pi}{8}, \frac{12\pi}{8}]$

$\tan(2x)=1$ $\quad 2x=\frac{\pi}{4}+2\pi n$ $\quad x=\frac{\pi}{8}+\frac{8\pi n}{8}$

$2x=\frac{5\pi}{4}+2\pi n$ $\quad x=\frac{5\pi}{8}+\frac{8\pi n}{8}$

n=	-1	0	1
	$\cancel{-\frac{7\pi}{8}}$	$\frac{\pi}{8}$	$\frac{9\pi}{8}$
	$-\frac{3\pi}{8}$	$\frac{5\pi}{8}$	$\cancel{\frac{13\pi}{8}}$

$\boxed{n=-\frac{3\pi}{8}, \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}}$

⑧ $16=-9\sin(7x)-4$ in $[-2\pi, \frac{9\pi}{4}]$

$\sin(7x)=-\frac{20}{9}$ $\quad \boxed{\text{no solution}}$ $\quad -1<\sin(x)<1$

⑨ $\sqrt{3}\tan\left(\frac{t}{4}\right)+5=4$ in $[0,4\pi] \to [0,\frac{24\pi}{6}]$

$\tan\left(\frac{t}{4}\right)=-\frac{1}{\sqrt{3}}$ $\quad \frac{t}{4}=\frac{5\pi}{6}+2\pi n$ $\quad t=\frac{20\pi}{6}+\frac{48\pi n}{6}$

$\frac{t}{4}=\frac{11\pi}{6}+2\pi n$ $\quad t=\frac{44\pi}{6}+\frac{48\pi n}{6}$

n=	-1	0
	$\cancel{}$	$\frac{20\pi}{6}$
	$\cancel{}$	$\cancel{}$

$\boxed{t=\frac{10\pi}{3}}$

⑩ $\sqrt{3}\csc(9z)-7=-5$ in $[-\frac{\pi}{3}, \frac{4\pi}{9}] \to [-\frac{9\pi}{27}, \frac{12\pi}{27}]$

$\csc(9z)=\frac{2}{\sqrt{3}}$ $\quad 9z=\frac{\pi}{3}+2\pi n$ $\quad z=\frac{\pi}{27}+\frac{6\pi n}{27}$

$\sin(9z)=\frac{\sqrt{3}}{2}$ $\quad 9z=\frac{2\pi}{3}+2\pi n$ $\quad z=\frac{2\pi}{27}+\frac{6\pi n}{27}$

n=	-1	0	1
	$-\frac{5\pi}{27}$	$\frac{\pi}{27}$	$\frac{7\pi}{27}$
	$-\frac{4\pi}{27}$	$\frac{2\pi}{27}$	$\frac{8\pi}{27}$

$=z$

(11) $1-14\cos(2x/5)=-6$ in $[5\pi, 40\pi/3] \to \left[\frac{30\pi}{6}, \frac{80\pi}{6}\right]$

$n = 1 \quad 2$

$\cos(2x/5) = 1/2$

$\frac{2x}{5} = \pi/3 + 2\pi n \qquad x = 5\pi/6 + \frac{30\pi n}{6} \qquad \frac{35\pi}{6} \quad \frac{65\pi}{6}$

$\frac{2x}{5} = 5\pi/3 + 2\pi n \qquad x = \frac{25\pi}{6} + \frac{30\pi n}{6} \qquad \frac{55\pi}{6} \quad \cancel{\frac{85\pi}{6}}$

$\boxed{n = \frac{35\pi}{6}, \frac{55\pi}{6}, \frac{65\pi}{6}}$

(12) $15 = 17 + 4\cos\left(\frac{y}{7}\right)$ in $[10\pi, 15\pi] \to [30\pi/3, 45\pi/3]$

$n = 0$

$\cos(y/7) = -1/2$

$y/7 = 2\pi/3 + 2\pi n \qquad y = \frac{14\pi}{3} + \frac{42\pi n}{3} \qquad 14\pi/3$

$\frac{y}{7} = 4\pi/3 + 2\pi n \qquad y = \frac{28\pi}{3} + \frac{42\pi n}{3} \qquad 28\pi/3$

$\boxed{\text{no solutions in the given interval}}$

see extra Sheet

Review : Solving Trig Equations with Calculators, Part I

Find the solution(s) to the following equations. If an interval is given then find only those solutions that are in the interval. If no interval is given then find all solutions to the equation. These will require the use of a calculator so use at least 4 decimal places in your work.

1. $7\cos(4x)+11=10$

2. $6+5\cos\left(\frac{x}{3}\right)=10$ in $[0,38]$

3. $3=6-11\sin\left(\frac{t}{8}\right)$

4. $4\sin(6z)+\frac{13}{10}=-\frac{3}{10}$ in $[0,2]$

5. $9\cos\left(\frac{4z}{9}\right)+21\sin\left(\frac{4z}{9}\right)=0$ in $[-10,10]$

6. $3\tan\left(\frac{w}{4}\right)-1=11-2\tan\left(\frac{w}{4}\right)$ in $[-50,0]$

7. $17-3\sec\left(\frac{z}{2}\right)=2$ in $[20,45]$

8. $12\sin(7y)+11=3+4\sin(7y)$ in $\left[-2,-\frac{1}{2}\right]$

9. $5-14\tan(8x)=30$ in $[-1,1]$

10. $0=18+2\csc\left(\frac{t}{3}\right)$ in $[0,5]$

11. $\frac{1}{2}\cos\left(\frac{x}{8}\right)+\frac{1}{4}=\frac{2}{3}$ in $[0,100]$

12. $\frac{4}{3}=1+3\sec(2t)$ in $[-4,6]$

Review : Solving Trig Equations with Calculators, Part II

Find all the solution(s) to the following equations. These will require the use of a calculator so use at least 4 decimal places in your work.

1. $3-14\sin\left(12t+7\right)=13$

2. $3\sec\left(4-9z\right)-24=0$

3. $4\sin\left(x+2\right)-15\sin\left(x+2\right)\tan\left(4x\right)=0$

4. $3\cos\left(\frac{3y}{7}\right)\sin\left(\frac{y}{2}\right)+14\cos\left(\frac{3y}{7}\right)=0$

5. $7\cos^2\left(3x\right)-\cos\left(3x\right)=0$

6. $\tan^2\left(\frac{w}{4}\right)=\tan\left(\frac{w}{4}\right)+12$

7. $4\csc^2\left(1-t\right)+6=25\csc\left(1-t\right)$

8. $4y\sec\left(7y\right)=-21y$

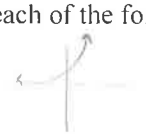
9. $10x^2\sin\left(3x+2\right)=7x\sin\left(3x+2\right)$

10. $\left(2t-3\right)\tan\left(\frac{6t}{11}\right)=15-10t$

Review : Exponential Functions

Sketch the graphs of each of the following functions.

1. $f\left(x\right)=3^{1+2x}$



2. $h\left(x\right)=2^{3-\frac{x}{4}}-7$

Solving Trig Eq. w/ Calculator - Part I

① $7\cos(4x)+11=10$ 1.7141

$\cos(4x) = -1/7$

$4x = \cos^{-1}(-1/7)$

$4x = 1.7141 + 2\pi n \rightarrow \boxed{x = 0.4285 + \frac{\pi n}{2}}$

$2\pi - 1.7141 \quad 4x = 4.5691 + 2\pi n \rightarrow \boxed{x = 1.1423 + \pi n/2}$

② $6 + 5\cos\left(\frac{x}{3}\right) = 10$ in $[0, 38]$

multiply by 3

$\cos\left(\frac{x}{3}\right) = 4/5$

$\cos^{-1}(4/5) = .6435$

$x/3 = .6435 + 2\pi n$

$x/3 = 5.6397 + 2\pi n$

$x = 1.9305 + 18.8496n$

$x = 16.9191 + 18.8496n$

n=	0	1
	1.9305	20.7801
	16.9191	35.7687

③ $3 = 6 - 11\sin(t/8)$

$\sin(t/8) = 3/11$

$\sin^{-1}(3/11) = .2762$

$\pi - .2762 = 2.8654$

$\boxed{t = 2.2096 + 16\pi n}$

$\boxed{t = 22.9232 + 16\pi n}$

④ $4\sin(6z) + 13/10 = -3/10$ in $[0, 2]$ $\frac{\pi n}{3}$

$\sin(6z) = -2/5$

$\sin^{-1}(-2/5) = -.4115$

$\pi + 0.4115 = 3.5531$

$z = -0.0686 + 1.0472n$

$z = 0.5922 + 1.0472n$

n=	0	1
	~~-0.0686~~	.9786
	0.5922	1.6394

$z = $ 0.5922, 0.9786, 1.6394

⑤ $9\cos\left(\frac{4z}{9}\right) + 21\sin\left(\frac{4z}{9}\right) = 0$ in $[-10, 10]$

$21\sin\left(\frac{4z}{9}\right) = -9\cos\left(\frac{4z}{9}\right)$ $\times 9/4$

$\tan\left(\frac{4z}{9}\right) = -3/7$

$\tan^{-1}(-3/7) = -.4049$

$\pi - 0.4049 = 2.7367$

change to $2\pi + -0.4049 = 5.8783$

positive angles only

$z = 13.2262 + 14.1372n$

$z = 6.1576 + 14.1372n$

n=	-1	0
	-0.911	~~13.2262~~
	-7.9796	6.1576

$z = $ -0.9110, -7.9796, 6.1576

⑥ $3 + \tan\left(\frac{w}{4}\right) - 1 = 11 - 2\tan\left(\frac{w}{4}\right)$ in $[-50, 0]$

$5\tan(w/4) = 12$

$\tan^{-1}(12/5) = 1.1760$

$\pi + 1.1760 = 4.3176$

(x4) $w = 4.704 + 25.1327n$

$w = 17.2704 + 25.1327n$

n=	-2	-1
	-45.5614	-20.4287
	-32.9950	-7.8623

⑦ $17 - 3\sec\left(\frac{z}{2}\right) = 2$ in $[20, 45]$

$\sec(z/2) = 5$

$\cos(z/2) = 1/5$

$\cos^{-1}(1/5) = 1.3694$

$2\pi - 1.3694 = 4.9138$

(x2) $z = 2.7388 + 12.5664n$

$z = 9.8276 + 12.5664n$

n=	1	2	3	4
	X	27.8716	40.4380	X
	22.3940	34.9604	X	

(8) $12\sin(7y)+11=3+4\sin(7y)$ in $[-2,-\frac{1}{2}]$ → Don't forget π is part of the answer

$8\sin(7y)=-8$ $\quad y=\frac{3\pi}{14}+\frac{4\pi n}{14}$ $\quad n=-2 \quad -1$

$\sin^{-1}(-1)=\frac{3\pi}{2}$ or $\quad \boxed{-1.122} \quad \times$

(9) $5-14\tan(8x)=30$ in $[-1,1]$

$\tan(8x)=\frac{-25}{14}$

$\tan^{-1}(\frac{25}{14})=-1.0603$ or 5.2229

$\pi+(-1.0603)=2.0813$

(÷8)

$x=0.2602+0.7854n$

$x=0.6529+0.7854\pi$

n=	-1	0	1
	-0.5252	0.2602	×
	-0.9179	-0.1325	0.6529	×

(10) $0=18+2\csc(t/3)$ in $[0,5]$

$\csc(t/3)=-9$

$\sin(t/3)=-1/9$

$\sin^{-1}(-1/9)=-0.1113$ or 6.1719

$\pi+0.1113=3.2529$

(×3)

$t=9.7587+18.8496n$ none

$t=18.5157+18.8496n$ none

$\boxed{\text{no solutions in this interval}}$

(11) $\frac{1}{2}\cos(x/8)+\frac{1}{4}=\frac{2}{3}$ in $[0,100]$

$\cos(x/8)=5/6$

$\cos^{-1}(5/6)=0.5857$

$2\pi-0.5857=5.6975$

(×8)

$x=4.6856+50.2655n$

$x=45.58+50.2655n$

$\boxed{\begin{array}{l} x=4.6856 \\ 45.5800 \\ 54.9511 \\ 95.8455 \end{array}}$

(12) $4/3=1+3\sec(2t)$ in $[-4,6]$

$\sec(2t)=1/9$

$\cos(2t)=9$

$\boxed{\text{no solution}}$

3. $h(t) = 8 + 3\mathbf{e}^{2t-4}$

4. $g(z) = 10 - \frac{1}{4}\mathbf{e}^{-2-3z}$

Review : Logarithm Functions

Without using a calculator determine the exact value of each of the following.

1. $\log_3 81$ $\quad 3^x = 81 \quad x = 4$

2. $\log_5 125$ $\quad 5^x = 125 \quad x = 3$

3. $\log_2 \frac{1}{8}$ $\quad 2^x = \frac{1}{8} \quad x = -3$

4. $\log_{\frac{1}{4}} 16$ $\quad \frac{1}{4}^x = 16 \quad x = -2$

5. $\ln \mathbf{e}^4$ $\quad 4$

6. $\log \frac{1}{100}$ $\quad 10^x = \frac{1}{100} \quad x = -2$

Write each of the following in terms of simpler logarithms

7. $\log\left(3x^4 y^{-7}\right)$ $\quad \log 3 + 4\log x - 7\log y$

8. $\ln\left(x\sqrt{y^2+z^2}\right)$ $\quad \ln x + \frac{1}{2}\ln(y^2+z^2)$

9. $\log_4\left(\frac{x-4}{y^2\sqrt[5]{z}}\right)$ $\quad \log_4(x-4) - [\log_4 y^2 + \log_4 \sqrt[5]{z}] = \log_4(x-4) - 2\log_4 y - \frac{1}{5}\log_4 z$

Combine each of the following into a single logarithm with a coefficient of one.

10. $2\log_4 x + 5\log_4 y - \frac{1}{2}\log_4 z$ $\log_4\left(\frac{x^2 y^5}{\sqrt{z}}\right)$

11. $3\ln(t+5) - 4\ln t - 2\ln(s-1)$ $\ln\left(\frac{(t+5)^3}{t^4(s-1)^2}\right)$

12. $\frac{1}{3}\log a - 6\log b + 2$ $\log\frac{100\sqrt[3]{a}}{b^6}$ $\log_{10} 100 = 2$

Use the change of base formula and a calculator to find the value of each of the following.

13. $\log_{12} 35$ $\frac{\ln 35}{\ln 12} = 1.43077731$

14. $\log_{\frac{2}{3}} 53$ $\frac{\ln 53}{\ln 2/3} = -9.79194469$

Review : Exponential and Logarithm Equations

For problems 1 – 10 find all the solutions to the given equation. If there is no solution to the equation clearly explain why.

no domain restrictions on e^x

1. $12 - 4\mathbf{e}^{7+3x} = 7$ $e^{7+3x} = \frac{5}{4}$ $7+3x = \ln(5/4)$ $x = \frac{\ln(5/4) - 7}{3}$ $\boxed{x = -2.25895}$

2. $1 = 10 - 3\mathbf{e}^{z^2-2z}$ $e^{z^2-2z} = 3$ $z^2 - 2z = \ln(3)$ $\boxed{Q}$ $\frac{2 \pm \sqrt{4 - 4(1)(\ln 3)}}{2} = 1 \pm \sqrt{1 + \ln(3)}$ $\boxed{z = -0.4487, 2.4487}$

3. $2t - t\mathbf{e}^{6t-1} = 0$ $t(2 - e^{6t-1}) = 0$ $e^{6t-1} = 2$ $6t - 1 = \ln(2)$ $t = \frac{\ln(2)+1}{6} = 0.2822$ $\boxed{t = 0 \quad t = 0.2822}$

4. $4x + 1 = (12x+3)\mathbf{e}^{x^2-2}$ $(4x+1)(1 - 3e^{x^2-2}) = 0$ $e^{x^2-2} = \frac{1}{3}$ $x^2 - 2 = \ln(1/3)$ $x^2 = \ln(1/3) + 2$ $x = \pm\sqrt{\ln(1/3)+2}$ $\boxed{x = -1/4 \quad x = \pm 0.9494}$

5. $2\mathbf{e}^{3y+8} - 11\mathbf{e}^{5-10y} = 0$ $2e^{3y+8} = 11e^{5-10y}$ $e^{3y+8-(5-10y)} = 11/2$ $e^{13y+3} = 11/2$ $13y + 3 = \ln(11/2)$ $y = \frac{\ln(11/2) - 3}{13}$ $\boxed{y = -0.09963}$

6. $14\mathbf{e}^{6-x} + \mathbf{e}^{12x-7} = 0$ $14e^{6-x} = -1e^{12x-7}$ $e^{6-x-(12x-7)} = -1/14$ $13 - 13x = \ln(-1/14)$ no Solution $\ln(-1/14)$ DNE

7. $1 - 8\ln\left(\frac{2x-1}{7}\right) = 14$ $\ln\left(\frac{2x-1}{7}\right) = \frac{-13}{8}$ $\frac{2x-1}{7} = e^{-13/8}$ $x = \frac{7e^{-13/8}+1}{2} = 1.1892$ ck in original $\left(\frac{2x-1}{7}\right)$ $\boxed{x = 1.1892}$

8. $\ln(y-1) = 1 + \ln(3y+2)$ $\ln(y-1) - \ln(3y+2) = 1$ $\ln\left(\frac{y-1}{3y+2}\right) = 1$ $\frac{y-1}{3y+2} = e^1$ $y - 1 = e(3y+2)$ $y - 1 = 3ey + 2e$ $y - 3ey = 1 + 2e$ $y(1 - 3e) = 1 + 2e$ $y = \frac{1+2e}{1-3e} = -0.8996$ ck: $\cancel{(y-1)}$ ck: $\cancel{(3y+2)}$

no Solution

9. $\log(w)+\log(w-21)=2$ $\quad \log w(w-21)=2 \quad w(w-21)=10^2 \quad w^2-21w-100=0 \quad (w-25)(w+4)=0 \quad w=25$ ✓ $\cancel{w=-4}$ ck in both!!

10. $2\log(z)-\log(7z-1)=0$ $\quad \log\frac{z^2}{7z-1}=0 \quad \frac{z^2}{7z-1}=1 \quad z^2=7z-1 \quad z^2-7z+1=0$ Q $\frac{7\pm\sqrt{49-4(1)(1)}}{2}$ $\boxed{z=0.1459}$ ✓ $\boxed{z=6.8541}$ ✓

11. $16=17^{t-2}+11$ $\quad 5=17^{t-2} \quad \ln(5)=\ln(17^{t-2}) \quad \ln(5)=(t-2)\ln 17 \quad t-2=\frac{\ln(5)}{\ln(17)}$ $\boxed{t=2.5681}$ no restriction

12. $2^{3-8w}-7=11$ $\quad 2^{3-8w}=18 \quad \ln(2^{3-8w})=\ln(18) \quad (3-8w)\ln(2)=\ln(18) \quad 3-8w=\frac{\ln(18)}{\ln(2)} \quad w=\frac{\frac{\ln(18)}{\ln(2)}-3}{-8}$ $\boxed{w=-0.1462}$

Compound Interest. If we put P dollars into an account that earns interest at a rate of r (written as a decimal as opposed to the standard percent) for t years then,

a. if interest is compounded m times per year we will have,

$$A=P\left(1+\frac{r}{m}\right)^{tm}$$

dollars after t years.

b. if interest is compounded continuously we will have,

$$A=P\mathbf{e}^{rt}$$

dollars after t years.

13. We have \$10,000 to invest for 44 months. How much money will we have if we put the money into an account that has an annual interest rate of 5.5% and interest is compounded
(a) quarterly (b) monthly (c) continuously

14. We are starting with \$5000 and we're going to put it into an account that earns an annual interest rate of 12%. How long should we leave the money in the account in order to double our money if interest is compounded
(a) quarterly (b) monthly (c) continuously

Exponential Growth/Decay. Many quantities in the world can be modeled (at least for a short time) by the exponential growth/decay equation.

$$Q=Q_0\mathbf{e}^{kt}$$

If k is positive then we will get exponential growth and if k is negative we will get exponential decay.

15. A population of bacteria initially has 250 present and in 5 days there will be 1600 bacteria present.
(a) Determine the exponential growth equation for this population.
(b) How long will it take for the population to grow from its initial population of 250 to

a population of 2000?

16. We initially have 100 grams of a radioactive element and in 1250 years there will be 80 grams left.
   (a) Determine the exponential decay equation for this element.
   (b) How long will it take for half of the element to decay?
   (c) How long will it take until there is only 1 gram of the element left?

TALK ABOUT THESE & SHOW GRAPH SOLUTIONS GIVE JOKE SHEET

Review : Common Graphs

Without using a graphing calculator sketch the graph of each of the following.

1. $y = \frac{4}{3}x - 2$

2. $f(x) = |x-3|$

3. $g(x) = \sin(x) + 6$ vertical shift

4. $f(x) = \ln(x) - 5$ vertical shift

5. $h(x) = \cos\left(x + \frac{\pi}{2}\right)$ phase shift

6. $h(x) = (x-3)^2 + 4$ horiz shift vert shift

7. $W(x) = \mathbf{e}^{x+2} - 3$

8. $f(y) = (y-1)^2 + 2$

9. $R(x) = -\sqrt{x}$

10. $g(x) = \sqrt{-x}$

11. $h(x) = 2x^2 - 3x + 4$

12. $f(y) = -4y^2 + 8y + 3$ left

13. $(x+1)^2 + (y-5)^2 = 9$

$x^2 - 4x + 4 + y^2 - 6y + \cancel{2}\ 87 + 4 + 9$

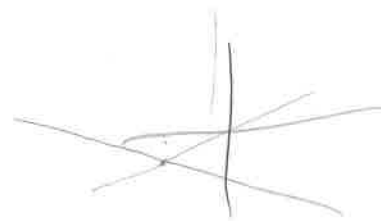
14. $x^2 - 4x + y^2 - 6y - 87 = 0$ $(x-2)^2 + (y-3)^2 = 100$

15. $25(x+2)^2 + \dfrac{y^2}{4} = 1$ $\dfrac{(x+2)^2}{1/25} + \dfrac{y^2}{4} = 1$ ellipse $a = \frac{1}{5}$ $b = 2$ center $(-2, 0)$

16. $x^2 + \dfrac{(y-6)^2}{9} = 1$ ellipse $a = 1$ $b = 3$ center $(0, 6)$

17. $\dfrac{x^2}{36} - \dfrac{y^2}{49} = 1$ hyperbola center $(0,0)$ opens right/left $(x^2 - y^2)$ vertices $(-6,0)$ $(6,0)$
asymptotes $\pm 7/6$

18. $\dfrac{(y+2)^2}{1} - \dfrac{(x+4)^2}{16} = 1$ hyperbola center $\overset{x}{(-4},\overset{y}{-2)}$ opens up/down $(y^2 - x^2)$
asymptotes $\pm 1/4$ vertices $(-4,-1)$ $(-4,-3)$



hyperbolas $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

asymptotes: $\pm b/a$

vertices:

center + a
center − a
for right/left

center + b
center − b
for up/down

Limits

Introduction

Here are a set of practice problems for the Limits chapter of my Calculus I notes. If you are viewing the pdf version of this document (as opposed to viewing it on the web) this document contains only the problems themselves and no solutions are included in this document. Solutions can be found in a number of places on the site.

7. If you'd like a pdf document containing the solutions go to the note page for the section you'd like solutions for and select the download solutions link from there. Or,

8. Go to the download page for the site http://tutorial.math.lamar.edu/download.aspx and select the section you'd like solutions for and a link will be provided there.

9. If you'd like to view the solutions on the web or solutions to an individual problem you can go to the problem set web page, select the problem you want the solution for. At this point I do not provide pdf versions of individual solutions, but for a particular problem you can select "Printable View" from the "Solution Pane Options" to get a printable version.

Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section.

Here is a list of topics in this chapter that have practice problems written for them.

Tangent Lines and Rates of Change
The Limit
One-Sided Limits
Limit Properties
Computing Limits
Infinite Limits
Limits At Infinity, Part I
Limits At Infinity, Part II
Continuity
The Definition of the Limit – Problems for this section have not yet been written.