College Algebra CHS

Name Key

Review for Final Exam

Date____________

Simplify. Your answer should contain only positive exponents.

1) $\dfrac{2y^3z^2}{(zx^2y^2)^{-2}}$ $= 2y^3z^2 z^2x^4y^4 = \boxed{2x^4y^7z^4}$

2) $\dfrac{p^{-1}}{2nm^{-4}p^2 \cdot pm^2n^3}$ $= \dfrac{m^4}{2np^2pm^2n^3p} = \dfrac{m^2}{2n^4p^4}$

3) $-\dfrac{2j^3k^2 \cdot 2h^{-2}j^{-4}k^{-2}}{(2h^{-2}k^{-3})^2}$ $= -\dfrac{4h^4k^6}{4h^2j} = \boxed{\dfrac{h^2k^6}{j}}$

$4h^{-4}k^{-6}$

4) $\dfrac{4x^{\frac{1}{4}}}{2x^{\frac{2}{3}}}$ $= \boxed{\dfrac{2}{x^{5/12}}}$

5) $(49x^4)^{\frac{3}{2}}$ $= (7x^2)^3 = \boxed{343x^6}$

6) $(45x^0y^3)^{-3}$ $= \boxed{\dfrac{1}{91125y^9}}$

Simplify. Your answer should contain only positive exponents.

7) $\sqrt{72x}$ $\boxed{6\sqrt{2x}}$

8) $\sqrt{256m^3n^3p}$ $\boxed{16mn\sqrt{mnp}}$

9) $\sqrt[4]{32x}$ $\boxed{2\sqrt[4]{2x}}$

10) $\sqrt[3]{1000x^3y^8z^6}$ $\boxed{10xy^2z^2\sqrt[3]{y^2}}$

Factor each completely.

11) $y^2x^2 + 2y^3x$ $xy^2(x+2y)$

12) $x^3 - 7x^2y$ $x^2(x-7y)$

13) $5x^4 - 26x^3 + 24x^2$ $x^2(5x-26x+24)$

$x^2(5x-6)(x-4)$

14) $27x^3 + 64$ $(3x+4)(9x^2-12x+16)$

Solve each equation by factoring.

15) $3b^2 + 25b + 8 = 0$

$(x+8)(3x+1)=0$

$x=-8$

$x=-1/3$

Solve each equation with the quadratic formula.

16) $3v^2 + 11v - 34 = 0$

$\frac{-11 \pm \sqrt{11^2 - 4(3)(-34)}}{2(3)} = \frac{-11 \pm \sqrt{529}}{6} = \frac{-11 \pm 23}{6}$

$x = 2$

$x = -\frac{17}{6}$

Evaluate each function.

17) $w(n) = n^3 + n$; Find $w(n + 3)$

$(n+3)^3 + (n+3)$

$(n+3)(n^2 + 6n + 9) + n + 3$

$n^3 + 6n^2 + 9n + 3n^2 + 18n + 27 + n + 3$

$\boxed{n^3 + 9n^2 + 28n + 30}$

Find the difference quotient for the following:

18) $g(x) = x^2 - 3x$

$\frac{(x+h)^2 - 3(x+h) - (x^2 - 3x)}{h} = \frac{\cancel{x^2} + 2hx + h^2 - \cancel{3x} - 3h - \cancel{x^2} + \cancel{3x}}{h} = \frac{2hx - 3h}{h}$

$= \frac{\cancel{h}(2x-3)}{\cancel{h}} = \boxed{2x-3}$

Find all roots.

19) $2x^4 - 17x^2 + 30 = 0$

$(x^2 - 6)(2x^2 - 5) = 0$

$x^2 = 6$

$\boxed{x = \pm\sqrt{6}}$

$2x^2 = 5$

$x^2 = 5/2$

$x = \pm\frac{\sqrt{5}}{\sqrt{2}} = \boxed{\frac{\pm\sqrt{10}}{2}}$

Find the domain and range of the following functions.

20) $M(t) = 4t^2 - 5t + 2$

$\frac{-(-5)}{2(4)} = \frac{5}{8}$

$4\left(\frac{5}{8}\right)^2 - 5\left(\frac{5}{8}\right) + 2 = \frac{7}{16}$

D: $\mathbb{R}$

R: $\left[\frac{7}{16}, \infty\right)$

21) $f(a) = \sqrt{3 - a^2} + 2$

D: $[-\sqrt{3}, \sqrt{3}]$ R: $[2, \infty)$

$3 - a^2 \geq 0$

$-a^2 \geq -3$

$a^2 \leq 3$

$-3 \leq a^2 \leq 3$

$-\sqrt{3} \leq a \leq \sqrt{3}$

$\sqrt{3-a^2} + 2$

$\uparrow$

0 or +

Find the domain of the following functions.

22) $h(x) = \frac{12}{x^2 + 4x + 3}$

$x \neq -3, x \neq -1$

$x^2 + 4x + 3 \neq 0$

$(x+3)(x+1) \neq 0$

$\boxed{(-\infty, -3) \cup (-3, -1) \cup (-1, \infty)}$

23) $b(t) = \sqrt{5x - 1}$

$\boxed{\left(\frac{1}{5}, \infty\right)}$

$5x - 1 > 0$

$5x > 1$

$x > \frac{1}{5}$

Graph the piecewise function. Find all requested limits.

24) $f(x) = \begin{cases} 3x-6, & x<-3 \\ x^2+16, & -3 \le x \le 2 \\ 4, & x>2 \end{cases}$

a. $\lim\limits_{x\to -6} f(x)$ $3(-6)-6 = \boxed{-24}$

b. $\lim\limits_{x\to -3} f(x)$ -3^-: $3(-3)-6=-15$ -3^+: $(-3)^2+16=25$ $\boxed{\text{DNE}}$

c. $\lim\limits_{x\to 2} f(x)$ 2^-: $2^2+16=20$ 2^+: 4 $\boxed{\text{DNE}}$

d. $\lim\limits_{x\to 7} f(x)$ $\boxed{4}$

Evaluate each limit.

25) $\lim\limits_{x\to -3} -\dfrac{x+3}{x^2+5x+6} = \dfrac{0}{0}$

$-\dfrac{\cancel{x+3}}{(\cancel{x+3})(x+2)} = \dfrac{-1}{x+2} = -\dfrac{1}{-1} = \boxed{1}$

26) $\lim\limits_{x\to -1^-} \dfrac{1}{x^2-1} = \dfrac{1}{1-1} = \dfrac{1}{0}$ asymptote $= \boxed{\infty}$

check left $\dfrac{1}{(-1.01)^2-1} = \dfrac{1}{.0201} = 49.75$ check graph

27) $\lim\limits_{x\to \frac{\pi}{2}} 2\tan(x) = \dfrac{2\sin x}{\cos x} = \dfrac{2}{0}$ = asymptote

check graph or unit circle $\lim\limits_{x\to \frac{\pi}{2}^-} = \infty$ $\lim\limits_{x\to \frac{\pi}{2}^+} = -\infty$ overall! $\boxed{\text{DNE}}$

28) $\lim\limits_{x\to 0} \dfrac{\tan 3x}{\cos 6x} = \dfrac{0}{1} = \boxed{0}$

29) $\lim\limits_{x\to \infty} \dfrac{x+2}{x^2+2x+2} = \dfrac{\cancel{x^2}\left(\frac{1}{x}+\frac{2}{x^2}\right)}{\cancel{x^2}\left(1+\frac{2}{x}+\frac{2}{x^2}\right)} = \dfrac{0}{1} = \boxed{0}$

30) $\lim\limits_{x\to \infty} \left(e^{3x}+3\right)$

$e^{\infty}+3 = \boxed{\infty}$

31) $\lim\limits_{x\to 3} \dfrac{(x-3)(\sqrt{x+6}+3)}{(\sqrt{x+6}-3)(\sqrt{x+6}+3)} = \dfrac{(x-3)(\sqrt{x+6}+3)}{x+6-9}$

$= \dfrac{\cancel{(x-3)}(\sqrt{x+6}+3)}{\cancel{(x-3)}} = \boxed{6}$

32) $\lim\limits_{x\to 0} \dfrac{1-\cos(5x)}{\cos(2x)-1} \cdot \dfrac{\cos(2x)+1}{\cos(2x)+1} = \dfrac{(1-\cos(5x))(\cos(2x)+1)}{\cos^2(2x)-1}$

Doesn't help

$\frac{0}{0}$ use L'hôpital's rule

$\lim\limits_{x\to 0} \dfrac{5\sin(5x)}{-2\sin(2x)} = \dfrac{0}{0}$

AGAIN

$\lim\limits_{x\to 0} \dfrac{25\cos(5x)}{-4\cos(2x)} = \boxed{\dfrac{25}{-4}}$

33) $\lim\limits_{x\to 0} \dfrac{\dfrac{1}{-3+x}+\dfrac{1}{3}}{x} = \dfrac{\dfrac{3}{3(-3+x)}+\dfrac{-3+x}{3(-3+x)}}{x}$

$= \dfrac{\dfrac{x}{3(-3+x)}}{x} = \dfrac{x}{3(-3+x)} \cdot \dfrac{1}{x} = \dfrac{1}{3(-3+x)} = \boxed{\dfrac{-1}{9}}$

Find the interval of continuity of the composition of functions.

34) $f(x) = (x-3)^5$ $\boxed{(-\infty, \infty)}$

$x-3$ continuous everywhere

u^5 continuous everywhere

35) $f(x) = \sqrt{5x+1}$ $\boxed{[0, \infty)}$

$5x+1$ continuous everywhere

$\sqrt{u}$ continuous $[0, \infty)$

Note all points of discontinuity for the function. Remove the discontinuity when possible by creating a piecewise function. If discontinuity cannot be removed, explain why.

36) $f(x) = \dfrac{x^4 - 16}{x^2 + x - 6} = \dfrac{(x^2-4)(x^2+4)}{(x+3)(x-2)} = \dfrac{\cancel{(x-2)}(x+2)(x^2+4)}{(x+3)\cancel{(x-2)}}$

Discontinuous at $x=2$, removable

Asymptote at $x=-3$ cannot be removed

$$\boxed{f(x) = \begin{cases} \dfrac{x^4-16}{x^2+x-6} & x \neq 2, -3 \\ \dfrac{32}{5} & x = 2 \end{cases}}$$

$$\lim_{x\to 2} \frac{(x+2)(x^2+4)}{(x+3)} = \frac{32}{5}$$

Identify all vertical, horizontal, and slant asymptotes as well as any holes. Give the equations of each asymptote.

37) $f(x) = \dfrac{x^2 - 5x + 4}{x^2 - 4x - 45} = \dfrac{(x-4)(x-1)}{(x-9)(x+5)}$

No holes
V. asymptotes: $x=9$, $x=-5$
H. asymptote: $x=1$
No slant asymptotes

Use the definition of the derivative to find the derivative of each function with respect to *x*.

38) $f(x) = 3x^2 - 3$

$$\lim_{h\to 0} \frac{3(x+h)^2 - 3 - (3x^2-3)}{h} = \frac{\cancel{3x^2} + 6xh + 3h^2 \cancel{-3} \cancel{-3x^2} \cancel{+3}}{h}$$

$$\lim_{h\to 0} = \frac{\cancel{h}(6x+3h)}{\cancel{h}} = 6x + 3h = \boxed{6x}$$

Differentiate each function with respect to x. Problems may contain constants a, b, and c.

39) $f(x) = cx^{4a} + \frac{4}{5}x^{2c} - \frac{3}{4}x^{\frac{3}{5}}$

$$\boxed{4acx^{4a-1} + \frac{8}{5}cx^{2c-1} - \frac{9}{20}x^{-2/5}}$$

Differentiate each function with respect to x.

40) $y = \left((3x^5 + 5)^3 - 3\right)^4$

$$4\left((3x^5+5)^3-3\right)^3 \cdot 3(3x^5+5)^2 \cdot 15x^4$$

$$\boxed{180x^4\left((3x^5+5)^3-3\right)^3(3x^5+5)^2}$$

41) $f(x) = \tan \sqrt[3]{-3x^5 - 1}$ $= \tan(-3x^5-1)^{1/3}$

$$\sec^2 \sqrt[3]{-3x^5-1} \cdot \frac{1}{3}(-3x^5-1)^{-2/3} \cdot -15x^4$$

$$\boxed{-5x^4 \sec^2 \sqrt[3]{-3x^5-1}\,(-3x^5-1)^{-2/3}}$$

42) $f(x) = \cos^{-1}(-4x^2)$

$$-\frac{1}{\sqrt{1-(-4x^2)^2}} \cdot -8x = \boxed{\frac{8x}{\sqrt{1-16x^4}}}$$

43) $f(x) = e^{5x^3}$ $\quad e^{5x^3} \cdot 15x^2 = \boxed{15x^2 e^{5x^3}}$

44) $y = \log_2 5x^3$ $\quad \frac{1}{5x^3 \ln 2} \cdot 15x^2 = \frac{15x^2}{5x^3 \ln 2} = \boxed{\frac{3}{x \ln 2}}$

45) $f(x) = 3^{5 + \log_4 3x^3}$ $\quad 3^{5+\log_4 3x^3} \ln 3 \cdot \frac{1}{3x^3 \ln 4} \cdot 9x^2 = \frac{9x^2 \ln 3 \cdot 3^{5+\log_4 3x^3}}{3x^3 \ln 4}$

$= \boxed{\frac{3 \ln 3 \cdot 3^{5+\log_4 3x^3}}{x \ln 4} \text{ or } \frac{3^{6+\log_4 3x^3} \ln 3}{x \ln 4}}$

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y.

46) $(3y^2 + 4)^2 = 5x$

$2(3y^2+4) \cdot 6y\,dy = 5\,dx$

$\frac{12y(3y^2+4)\,dy}{12y(3y^2+4)} = \frac{5\,dx}{12y(3y^2+4)}$

$dy = \frac{5}{12y(3y^2+4)}\,dx \Rightarrow \boxed{\frac{dy}{dx} = \frac{5}{12y(3y^2+4)}}$

For each problem, find the indicated derivative with respect to x.

47) $f(x) = 5x^3 - 2x^2 + 5x$ Find all derivatives.

$f'(x) = 15x^2 - 4x + 5$

$f''(x) = 30x - 4$

$f'''(x) = 30$

$f^{(4)}(x) = 0$

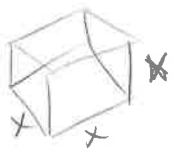
For each problem, find the instantaneous rate of change of the function at the given value.

48) $y = -x^2 + 1;\ \ -2$

$$y' = -2x \Big|_{-2} = \boxed{+4}$$

Solve each related rate problem.

49) A hypothetical cube shrinks so that the length of its sides are decreasing at a rate of 3 m/min. At what rate is the volume of the cube changing when the sides are 8 m each?

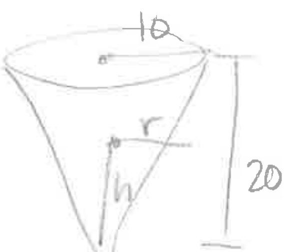


$V = x^3 \qquad \frac{dV}{dt} = ? \qquad \frac{dx}{dt} = -3\frac{m}{min} \qquad x = 8$

$$\frac{dV}{dt} = 3x^2\frac{dx}{dt}$$

$$\frac{dV}{dt} = 3(8)^2(-3) = \boxed{-576\ m^3/min}$$

50) A conical paper cup is 20 cm tall with a radius of 10 cm. The bottom of the cup is punctured so that the water leaks out at a rate of $\frac{9\pi}{4}$ cm³/sec. At what rate is the water level changing when the water level is 2 cm?



$\frac{dV}{dt} = -\frac{9\pi}{4}\frac{cm^3}{sec} \qquad \frac{dh}{dt} = ? \qquad h = 2cm$

$$\frac{r}{h} = \frac{10}{20}$$

$$20r = 10h$$

$$r = \frac{1}{2}h$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h$$

$$V = \frac{1}{3}\pi\frac{1}{4}h^2 h$$

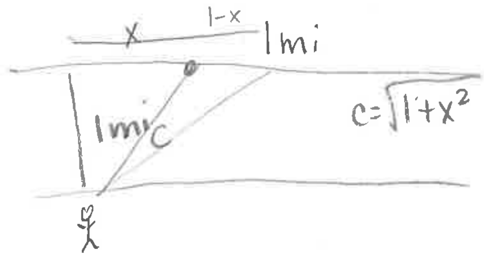
$$V = \frac{1}{12}\pi h^3$$

$$\frac{dV}{dt} = \frac{1}{4}\pi h^2\frac{dh}{dt}$$

$$-\frac{9\pi}{4} = \frac{1}{4}\pi(2)^2\frac{dh}{dt}$$

$$\boxed{-\frac{9}{4}\frac{cm}{sec}} = \frac{dh}{dt}$$

51) You are standing at the edge of a slow-moving river which is one mile wide and wish to return to your campground on the opposite side of the river. You can swim at 2 mph and walk at 3 mph. You must first swim across the river to any point on the opposite bank. From there, walk to the campground, which is one mile from the point directly across the river from where you start your swim. What route will take the least amount of time?



$c=\sqrt{1+x^2}$

$d=rt$ $\quad t=\frac{d}{r}$

swim: $t=\frac{d}{2}$ $\quad$ walk: $t=\frac{d}{3}$

$\boxed{\text{You should swim } \frac{4\sqrt{5}}{5} \text{ mi downshore, then walk.}}$

① Across, then walk

swim: $\frac{1}{2}$

walk: $\frac{1}{3}$ $= \frac{5}{6}$ hrs $\approx .8\overline{3}$

② swim

$1^2+1^2=c^2$

$2=c^2$

$c=\sqrt{2}$

$\frac{\sqrt{2}}{2} \approx .707$

③ Land somewhere, then walk.

$T=\frac{\sqrt{1+x^2}}{2}+\frac{1-x}{3}=\frac{1}{2}(1+x^2)^{1/2}+\frac{1}{3}(1-x)$

$T'=\frac{1}{4}(1+x^2)^{-1/2}(2x)+\frac{1}{3}\cdot(-1)=\frac{x}{2\sqrt{1+x^2}}-\frac{1}{3}$

$=\frac{3x-2\sqrt{1+x^2}}{6\sqrt{1+x^2}}=0$ ← never zero

$3x-2\sqrt{1+x^2}=0$

$(3x)^2=(2\sqrt{1+x^2})^2$

$9x^2=4(1+x^2)$

$9x^2=4+4x^2$

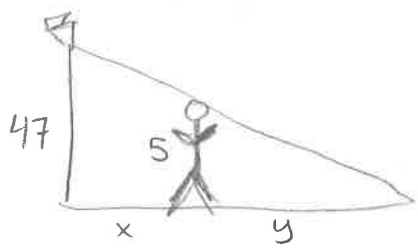
$5x^2=4$

$x^2=4/5$ $\quad x\approx .8944$

$T=\frac{\sqrt{1+(.8944)^2}}{2}+\frac{1-.8944}{3}\approx .706$

$.6708 + .0352$

52) A light shines from the top of a pole 47 ft high. A 5 ft person walks away from the light pole at a rate of 1.3 ft/sec. How fast is the distance from the light post to the tip of the shadow increasing? How fast is the shadow increasing?



$\frac{dx}{dt}=1.3$

$\frac{dy}{dt}=?$ $\quad \frac{dz}{dt}=?$

$\frac{5}{y}=\frac{47}{x+y}$

$5x+5y=47y$

$5x=42y$

$5\frac{dx}{dt}=42\frac{dy}{dt}$

$5(1.3)=42\frac{dy}{dt}$

$\frac{dy}{dt}\approx .1548$ ft/sec

$\boxed{\text{Shadow lengthening at a rate of .1548 ft/sec}}$

$\frac{5}{z-x}=\frac{47}{z}$

$5z=47(z-x)$

$5z=47z-47x$

$-42z=-47x$

$-42\frac{dz}{dt}=-47\frac{dx}{dt}$

$-42\frac{dz}{dt}=-47(1.3)$

$\frac{dz}{dt}\approx 1.4548$ ft/sec

$\boxed{\text{distance to tip of Shadow increasing at 1.4548 ft/sec}}$

For each problem, find all points of absolute minima and maxima on the given interval.

53) $y = -\dfrac{3}{x^2 - 9}$; $[-1, 1]$

$y' = \dfrac{0(x^2-9) + +3(2x)}{(x^2-9)^2}$

$6x = 0$ $\quad x^2 - 9 = 0$ not in interval
$x = 0$ $\quad x^2 = 9$ $\quad \cancel{x = \pm 3}$

$(0, \frac{1}{3})$
$(-1, \frac{3}{8})$
$(1, \frac{3}{8})$

$\boxed{\begin{array}{l}\text{Abs min } (0, \frac{1}{3}) \\ \text{Abs max } (-1, \frac{3}{8})\ (1, \frac{3}{8})\end{array}}$

Plug in original

Find the x-coordinates of critical points, then find the open intervals where the function is increasing and decreasing.

54) $h(s) = -s^4 + s^2 + 1$

$h'(s) = -4s^3 + 2s = 0$
$-2s(2s^2 - 1) = 0$
$-2s = 0 \quad 2s^2 - 1 = 0$
$s = 0 \quad 2s^2 = 1$
$s^2 = \frac{1}{2} \quad s = \pm\frac{\sqrt{2}}{2}$

$+ \quad - \quad + \quad -$
$-\frac{\sqrt{2}}{2} \quad 0 \quad \frac{\sqrt{2}}{2}$

$\boxed{\begin{array}{l}\text{Increasing } (-\infty, -\frac{\sqrt{2}}{2}), (0, \frac{\sqrt{2}}{2}) \\ \text{Decreasing } (-\frac{\sqrt{2}}{2}, 0), (\frac{\sqrt{2}}{2}, \infty)\end{array}}$

Critical pts: $0, -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}$

55) $g(w) = -\cot(w)$; $[-\pi, \pi]$

$g'(w) = \csc^2(w) = 0$ NO critical points
$\frac{1}{\sin^2(w)} = 0$
can't equal zero

$+ \quad +$
0

$\boxed{\begin{array}{l}\text{Increasing } (-\infty, 0), (0, \infty) \\ \text{Decreasing - no intervals}\end{array}}$

56) $h(s) = -\dfrac{2}{s^2 - 9}$

$\dfrac{0(s^2-9) + 2(2s)}{(s^2-9)^2} = 0$

$4s = 0 \quad s^2 - 9 = 0$
$s = 0 \quad s^2 = 9 \quad s = \pm 3$

$- \quad - \quad + \quad +$
$-3 \quad 0 \quad 3$

$\boxed{\begin{array}{l}\text{Increasing } (0, 3)(3, \infty) \\ \text{Decreasing } (-\infty, -3), (-3, 0)\end{array}}$

Critical pts: 0

For each problem, find the x-coordinates of all points of inflection and find the open intervals where the function is concave up and concave down.

57) $y = -x^3 + 2x^2 - 1$

$y' = -3x^2 + 4x$
$y'' = -6x + 4 = 0$
$-6x = -4$
$x = \frac{2}{3}$

$+ \quad -$
$\frac{2}{3}$

inflection pt: $\frac{2}{3}$

$\boxed{\begin{array}{l}\text{Concave up } (-\infty, \frac{2}{3}) \\ \text{Concave down } (\frac{2}{3}, \infty)\end{array}}$

58) $y = -2\cos(2x)$; $[-\pi, \pi]$

$y' = 4\sin(2x)$

$y'' = 8\cos(2x) = 0$

$\cos(2x) = 0$

$2x = \pi/2 + 2\pi n$

$2x = 3\pi/2 + 2\pi n$

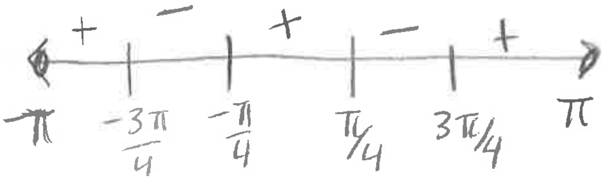
$x = \pi/4 + \pi n$

$x = 3\pi/4 + \pi n$

Test pts:

$-\pi, -\frac{3\pi}{4}, \frac{\pi}{4}$

$-\frac{\pi}{4}, \frac{3\pi}{4}, \pi$

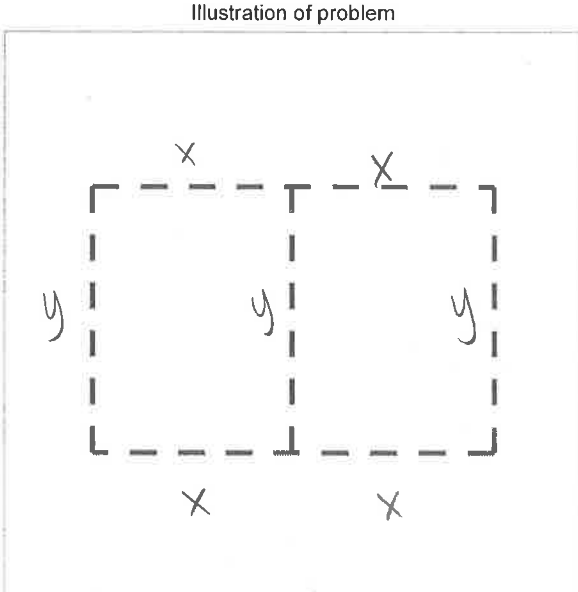


$\boxed{\begin{array}{l} \text{Concave up: } \left(-\pi, -\frac{3\pi}{4}\right), \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)\left(\frac{3\pi}{4}, \pi\right) \\ \text{Concave down: } \left(-\frac{3\pi}{4}, -\frac{\pi}{4}\right)\left(\frac{\pi}{4}, \frac{3\pi}{4}\right) \\ \text{Inflection points: } x = -\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4} \end{array}}$

Solve each optimization problem.

59) A rancher wants to construct two identical rectangular corrals using 200 ft of fencing. The rancher decides to build them adjacent to each other, so they share fencing on one side. What dimensions should the rancher use to construct each corral so that together, they will enclose the largest possible area?

Illustration of problem



Constraint

$4x + 3y = 200$

$3y = 200 - 4x$

$y = \frac{200 - 4x}{3}$

Maximize

$A = 2xy$

$A = 2x\left(\frac{200 - 4x}{3}\right) = \frac{400x}{3} - \frac{8x^2}{3}$

$A' = \frac{400}{3} - \frac{16}{3}x = 0$

$\frac{16}{3}x = \frac{400}{3}$

$x = 25$

$4(25) + 3y = 200$

$100 + 3y = 200$

$3y = 100$

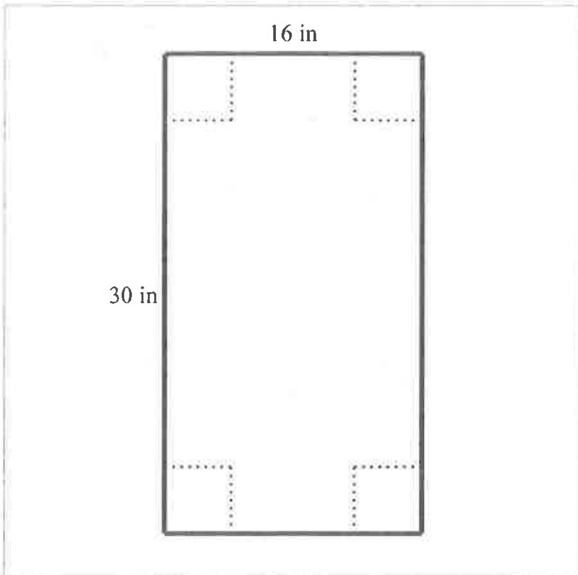
$y = \frac{100}{3}$

Dimensions of each corral:

$25 \times \frac{100}{3}$

60) A supermarket employee wants to construct an open-top box from a 16 by 30 in piece of cardboard. To do this, the employee plans to cut out squares of equal size from the four corners so the four sides can be bent upwards. What size should the squares be in order to create a box with the largest possible volume?

Illustration of problem



Maximize

$V = x(16-2x)(30-2x)$

$V = x(4x^2 - 92x + 480)$

$V = 4x^3 - 92x^2 + 480x$

$V' = 12x^2 - 184x + 480 = 0$

$4(x^2 - 46x + 120) = 0$

$4(x-12)(3x-10) = 0$

$\cancel{x = 12} \quad x = \frac{10}{3}$

$\boxed{\text{sides of square} = \frac{10}{3} \text{ in}}$

61) We want to construct a cylindrical can with a bottom but no top that will have a volume of 120π in^3. Determine the dimensions of the can that will minimize the amount of material needed to construct the can.

Constraint:

$V = \pi r^2 h = 120\pi$

$h = \frac{120\pi}{\pi r^2} = \frac{120}{r^2}$

$\boxed{\begin{array}{l}\text{radius} = 4.9324 \\ \text{height} = 4.9325\end{array}}$

Minimize

$SA = \pi r^2 + 2\pi r h$

$SA = \pi r^2 + 2\pi r\left(\frac{120}{r^2}\right)$

$SA = \pi r^2 + \frac{240\pi}{r}$

$SA' = 2\pi r - \frac{240\pi}{r^2} = 0$

$\frac{240\pi}{r^2} = 2\pi r$

$240\pi = 2\pi r^3$

$120 = r^3$

$r = 2\sqrt[3]{15} \approx 4.9324$

$h = \frac{120}{(4.9324)^2} = 4.9325$

Evaluate each limit using L'Hôpital's Rule.

62) $\lim_{x\to\infty} \frac{e^{2x}}{x} = \frac{\infty}{\infty}$ $\quad \frac{2e^{2x}}{1} = \boxed{\infty}$

63) $\lim_{x\to\infty} 4x\sin\frac{1}{x}$ $\quad \left(\frac{\sin\frac{1}{x}}{\frac{1}{4x}}\right)\frac{d}{dx}$ $= \frac{0}{0}$ $\quad = \frac{\cos\frac{1}{x}\cdot -x^{-2}}{-(4x)^{-2}\cdot 4} = \frac{16x^2\cos\frac{1}{x}}{4x^2}$

$\lim_{x\to\infty} = 4\cos\frac{1}{x} = 4\cdot 1 = \boxed{4}$

64) $\lim_{x\to -1}\left(\frac{3(x^2-1)}{\ln x^2}\right) = \frac{0}{0}$ $\quad 3x^2-3$ $\quad \frac{d}{dx}$ $\quad \frac{6x}{\frac{1}{x^2}\cdot 2x} = \frac{6x}{\frac{2}{x}} = 6x\cdot\frac{x}{2} = \frac{6x^2}{2} =$

$\lim_{x\to -1} 3x^2 = \boxed{3}$

65) $\lim_{x\to 0}\left(\frac{\sin(5x)}{3x}\right) = \frac{0}{0}$ $\quad \frac{d}{dx}$

$\frac{5\cos(5x)}{3} = \frac{5}{3}\cdot 1 = \boxed{\frac{5}{3}}$

66) $\lim_{x\to\frac{\pi}{2}} 3\cdot(\sin x)^{\tan 2x}$

$y = 3\sin x^{\tan 2x}$

$\frac{1}{3}y = \sin x^{\tan 2x}$

$\lim_{x\to\frac{\pi}{2}}$ $\boxed{\ln\frac{1}{3}y} = \tan 2x \ln\sin x = 0\cdot 0$ indeterminate

take limit

$\left(\frac{\ln\sin x}{\tan 2x}\right)\frac{d}{dx}$ $\frac{0}{0}$ $\quad \frac{\frac{1}{\sin x}\cdot\cos x}{2\sec^2(2x)} = \frac{\frac{\cos x}{\sin x}}{\frac{2}{\cos^2(2x)}} = \frac{\frac{0}{1}}{\frac{2}{1}} = \frac{0}{2} = 0$

$e^{\ln\frac{1}{3}y} = e^0$

$\frac{1}{3}y = 1$

$\boxed{y = 3}$

Evaluate each indefinite integral.

67) $\int (8x^3 - 15x^2 + 4)\, dx$

$\boxed{2x^4 - 5x^3 + 4x + C}$

68) $\int\left(-25x^4 - \frac{24\sqrt[5]{x}}{5} + 8x^{-5}\right) dx$ $\quad x^{1/5}$

$-5x^5 - \frac{24}{5}\cdot\frac{5}{6}x^{6/5} + \frac{8\cdot 1}{-4}x^{-4} + C$

$\boxed{-5x^5 - 4x^{6/5} - 2x^{-4} + C}$

69) $\int\left(\frac{10x^{\frac{3}{2}}}{2} + \frac{27x^{\frac{5}{4}}}{4} - 12x^{-5}\right) dx$

$\frac{10}{2}\cdot\frac{2}{5}x^{5/2} + \frac{27}{4}\cdot\frac{4}{9}x^{9/4} - \frac{12\cdot 1}{-4}x^{-4} + C$

$\boxed{2x^{5/2} + 3x^{9/4} + 3x^{-4} + C}$

70) $\int -\frac{5}{\csc x}\, dx = \int -5\sin x\, dx$

$-5\cdot -\cos x + C$

$\boxed{5\cos x + C}$

71) $\int 4\cot x\,dx = 4\int \frac{\cos x}{\sin x}\,dx$

$u = \sin x$

$du = \cos x\,dx$

$4\int \frac{1}{u}\,du$

$\boxed{4\ln|\sin x| + C}$

72) $\int 9x^2(3x^3 - 1)^4\,dx$ $\boxed{\frac{1}{5}(3x^3-1)^5 + C}$

$u = 3x^3 - 1$

$du = 9x^2\,dx$

$\int u^4\,du$

73) $4 \cdot \int (4x^5 - 1)^5 \cdot \overset{\cdot 1/4}{80x^4}\,dx$

$u = 4x^5 - 1$

$du = 20x^4\,dx$

$4\int u^5\,du$

$4 \cdot \frac{1}{6}u^6 + C = \boxed{\frac{2}{3}(4x^5-1)^6 + C}$

74) $\int \frac{(5 + \ln -x)^3}{x}\,dx$ $\boxed{\frac{1}{4}(5+\ln(-x))^4 + C}$

$u = 5 + \ln(-x)$

$du = \frac{1}{-x} \cdot -1 = \frac{1}{x}\,dx$

$\int u^3\,du$

75) $\int -\sec(-x)\tan(-x)\sec^5(-x)\,dx$

$(\sec(-x))^5$

$u = \sec(-x)$

$du = \sec(-x)\tan(-x) \cdot -1\,dx$

$\int u^5\,du$

$\boxed{\frac{1}{6}\sec^6(-x) + C}$

76) $3 \cdot \int \overset{\cdot 1/3}{12}\csc(-4x) \cdot \cot(-4x) \cdot \csc^5(-4x)\,dx$

$u = \csc(-4x)$

$du = -\csc(-4x)\cot(-4x) \cdot -4$

$= 4\csc(-4x)\cot(-4x)\,dx$

$3\int u^5\,du$

$3 \cdot \frac{1}{6}u^6 + C = \boxed{\frac{1}{2}\csc^6(-4x) + C}$

77) $\int (e^x - 4)^3 \cdot e^x\,dx$

$u = e^x - 4$

$du = e^x\,dx$

$\int u^3\,du$

$\boxed{\frac{1}{4}(e^x-4)^4 + C}$

78) $\int \frac{25x^4}{\sqrt{1 - 25x^{10}}}\,dx$

$\int \frac{25x^4}{\sqrt{1-(5x^5)^2}}$

$u = 5x^5$

$du = 25x^4\,dx$

$\int \frac{1}{\sqrt{1-u^2}}\,du$

$\sin^{-1}(u) + C$

$\boxed{\sin^{-1}(5x^5) + C}$

79) $\int \frac{10x}{25 + 25x^4}\,dx$

$\frac{.5}{25}\int \frac{\overset{\cdot 1/5}{10x}}{25(1+x^4)}\,dx$

$\int \frac{10x}{25(1+(x^2)^2)}\,dx$

$u = x^2$

$du = 2x\,dx$

$\frac{1}{5}\int \frac{1}{1+u^2}\,du$

$\frac{1}{5}\tan^{-1}(u) + C$

$\boxed{\frac{1}{5}\tan^{-1}(x^2) + C}$

Evaluate each definite integral.

80) $\int_{-2}^{1} (x^4 + x^3 - 2x^2 + 3)\, dx$

$\frac{1}{5}x^5 + \frac{1x^4}{4} - \frac{2}{3}x^3 + 3x \Big|_{-2}^{1} = \boxed{\frac{117}{20}}$

81) $\int_{-1}^{3} 4x^{\frac{1}{3}}\, dx$

$4 \cdot \frac{3}{4} x^{4/3} = 3x^{4/3} \Big|_{-1}^{3} =$

$\boxed{3(3)^{4/3} - 3(-1)^{4/3}}$

82) $\int_{-3}^{-2} -\frac{3}{x^2}\, dx = \int_{-3}^{-2} -3x^{-2} dx$

$3x^{-1} \Big|_{-3}^{-2}$

$\frac{3}{-2} + \frac{3}{+3} = \boxed{\frac{-1}{2}}$

83) $\int_{-1}^{0} -2e^x\, dx$

$-2e^x \Big|_{1}^{0}$

$\boxed{-2 - 2e}$

84) $\int_{-3}^{-2} -\frac{1}{x}\, dx$

$-\ln|x| \Big|_{-3}^{-2}$

$\boxed{-\ln(2) + \ln(3)}$

85) $\int_{-\frac{\pi}{6}}^{0} -2 \cdot \sec^2 x\, dx$

$-2\tan x \Big|_{-\frac{\pi}{6}}^{0}$

$0 + {}^{+}2\tan\left(\frac{-\pi}{6}\right)$

$2\left(\frac{-\sqrt{3}}{3}\right) = \boxed{\frac{-2\sqrt{3}}{3}}$

86) $\int_{-\frac{\pi}{4}}^{\frac{3\pi}{4}} -2 \cdot \cancel{\sec x}\ \sin x\, dx$

$-2 \cdot -\cos x \Big|_{-\frac{\pi}{4}}^{\frac{3\pi}{4}}$

$2\cos x \Big|_{-\pi/4}^{3\pi/4}$

$2 \cdot \frac{-\sqrt{2}}{2} - 2\frac{\sqrt{2}}{2} = \boxed{-2\sqrt{2}}$

College Algebra CHS   Name____________________

Review for Final Exam   Date____________

Simplify. Your answer should contain only positive exponents.

1) $\dfrac{2y^3z^2}{(zx^2y^2)^{-2}}$

2) $\dfrac{p^{-1}}{2nm^{-4}p^2 \cdot pm^2n^3}$

3) $-\dfrac{2j^3k^2 \cdot 2h^{-2}j^{-4}k^{-2}}{(2h^{-2}k^{-3})^2}$

4) $\dfrac{4x^{\frac{1}{4}}}{2x^{\frac{2}{3}}}$

5) $(49x^4)^{\frac{3}{2}}$

6) $(45x^0y^3)^{-3}$

Simplify. Your answer should contain only positive exponents.

7) $\sqrt{72x}$

8) $\sqrt{256m^3n^3p}$

9) $\sqrt[4]{32x}$

10) $\sqrt[3]{1000x^3y^8z^6}$

Factor each completely.

11) $y^2x^2 + 2y^3x$

12) $x^3 - 7x^2y$

13) $5x^4 - 26x^3 + 24x^2$

14) $27x^3 + 64$

Solve each equation by factoring.

15) $3b^2 + 25b + 8 = 0$

Solve each equation with the quadratic formula.

16) $3v^2 + 11v - 34 = 0$

Evaluate each function.

17) $w(n) = n^3 + n$; Find $w(n + 3)$

Find the difference quotient for the following:

18) $g(\mathrm{x}) = x^2 - 3x$

Find all roots.

19) $2x^4 - 17x^2 + 30 = 0$

Find the domain and range of the following functions.

20) $M(\mathrm{t}) = 4t^2 - 5t + 2$

21) $f(\mathrm{a}) = \sqrt{3 - a^2} + 2$

Find the domain of the following functions.

22) $\mathrm{h(x)} = \dfrac{12}{x^2 + 4x + 3}$

23) $\mathrm{b(t)} = \sqrt{5x - 1}$

Graph the piecewise function. Find all requested limits.

24) $f(x) = \begin{cases} 3x - 6, & x < -3 \\ x^2 + 16, & -3 \le x \le 2 \\ 4, & x > 2 \end{cases}$

a. $\lim_{x \to -6} f(x)$

b. $\lim_{x \to -3} f(x)$

c. $\lim_{x \to 2} f(x)$

d. $\lim_{x \to 7} f(x)$

Evaluate each limit.

25) $\lim_{x \to -3} -\frac{x + 3}{x^2 + 5x + 6}$

26) $\lim_{x \to -1^-} \frac{1}{x^2 - 1}$

27) $\lim_{x \to \frac{\pi}{2}} 2\tan(x)$

28) $\lim_{x \to 0} \frac{\tan 3x}{\cos 6x}$

29) $\lim_{x \to \infty} \frac{x + 2}{x^2 + 2x + 2}$

30) $\lim_{x \to \infty} \left(e^{3x} + 3\right)$

31) $\lim_{x \to 3} \frac{x - 3}{\sqrt{x + 6} - 3}$

32) $\lim_{x \to 0} \frac{1 - \cos(5x)}{\cos(2x) - 1}$

33) $\lim_{x \to 0} \frac{\frac{1}{-3 + x} + \frac{1}{3}}{x}$

Find the interval of continuity of the composition of functions.

34) $f(x) = (x - 3)^5$

35) $f(x) = \sqrt{5x + 1}$

Note all points of discontinuity for the function. Remove the discontinuity when possible by creating a piecewise function. If discontinuity cannot be removed, explain why.

36) $f(x) = \dfrac{x^4 - 16}{x^2 + x - 6}$

Identify all vertical, horizontal, and slant asymptotes as well as any holes. Give the equations of each asymptote.

37) $f(x) = \dfrac{x^2 - 5x + 4}{x^2 - 4x - 45}$

Use the definition of the derivative to find the derivative of each function with respect to x.

38) $f(x) = 3x^2 - 3$

Differentiate each function with respect to x. Problems may contain constants a, b, and c.

39) $f(x) = cx^{4a} + \frac{4}{5}x^{2c} - \frac{3}{4}x^{\frac{3}{5}}$

Differentiate each function with respect to x.

40) $y = ((3x^5 + 5)^3 - 3)^4$

41) $f(x) = \tan \sqrt[3]{-3x^5 - 1}$

42) $f(x) = \cos^{-1} -4x^2$

43) $f(x) = e^{5x^3}$

44) $y = \log_2 5x^3$

45) $f(x) = 3^{5 + \log_4 3x^3}$

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y.

46) $(3y^2 + 4)^2 = 5x$

For each problem, find the indicated derivative with respect to x.

47) $f(x) = 5x^3 - 2x^2 + 5x$ Find all derivatives.

For each problem, find the instantaneous rate of change of the function at the given value.

48) $y = -x^2 + 1;\quad -2$

Solve each related rate problem.

49) A hypothetical cube shrinks so that the length of its sides are decreasing at a rate of 3 m/min. At what rate is the volume of the cube changing when the sides are 8 m each?

50) A conical paper cup is 20 cm tall with a radius of 10 cm. The bottom of the cup is punctured so that the water leaks out at a rate of $\frac{9\pi}{4}$ cm³/sec. At what rate is the water level changing when the water level is 2 cm?

51) You are standing at the edge of a slow-moving river which is one mile wide and wish to return to your campground on the opposite side of the river. You can swim at 2 mph and walk at 3 mph. You must first swim across the river to any point on the opposite bank. From there, walk to the campground, which is one mile from the point directly across the river from where you start your swim. What route will take the least amount of time?

52) A light shines from the top of a pole 47 ft high. A 5 ft person walks away from the light pole at a rate of 1.3 ft/sec. How fast is the distance from the light post to the tip of the shadow increasing? How fast is the shadow increasing?

For each problem, find all points of absolute minima and maxima on the given interval.

53) $y = -\frac{3}{x^2 - 9}$; $[-1, 1]$

Find the x-coordinates of critical points, then find the open intervals where the function is increasing and decreasing.

54) $h(s) = -s^4 + s^2 + 1$

55) $g(w) = -\cot(w)$; $[-\pi, \pi]$

56) $h(s) = -\frac{2}{s^2 - 9}$

For each problem, find the x-coordinates of all points of inflection and find the open intervals where the function is concave up and concave down.

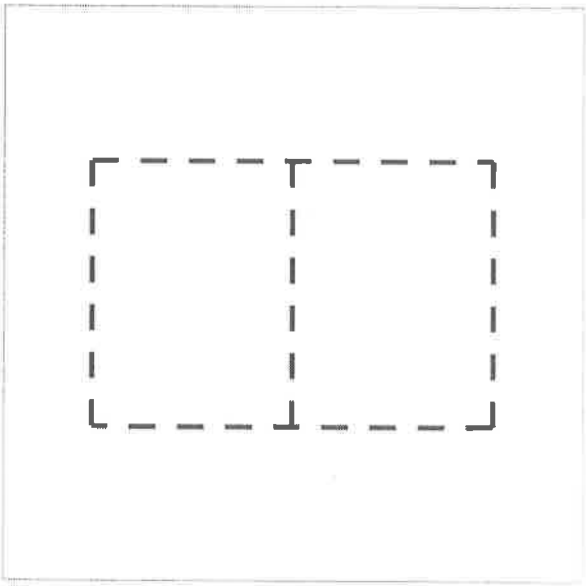
57) $y = -x^3 + 2x^2 - 1$

58) $y = -2\cos(2x)$; $[-\pi, \pi]$

Solve each optimization problem.

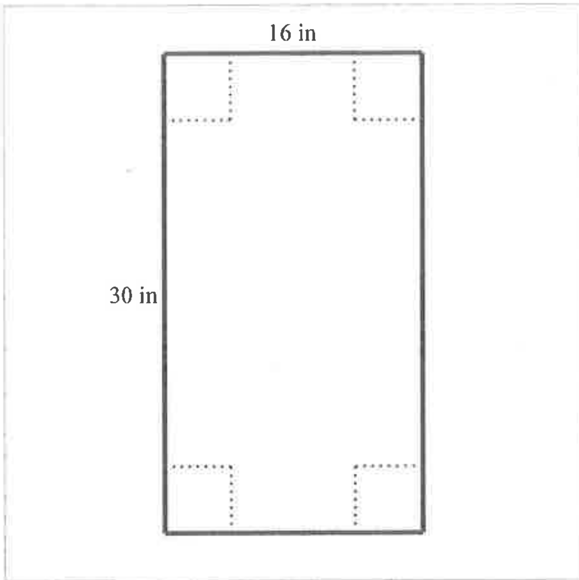
59) A rancher wants to construct two identical rectangular corrals using 200 ft of fencing. The rancher decides to build them adjacent to each other, so they share fencing on one side. What dimensions should the rancher use to construct each corral so that together, they will enclose the largest possible area?

Illustration of problem



60) A supermarket employee wants to construct an open-top box from a 16 by 30 in piece of cardboard. To do this, the employee plans to cut out squares of equal size from the four corners so the four sides can be bent upwards. What size should the squares be in order to create a box with the largest possible volume?

Illustration of problem



61) We want to construct a cylindrical can with a bottom but no top that will have a volume of 120π in^3. Determine the dimensions of the can that will minimize the amount of material needed to construct the can.

Evaluate each limit using L'Hôpital's Rule.

62) $\lim\limits_{x\to\infty} \dfrac{e^{2x}}{x}$

63) $\lim\limits_{x\to\infty} 4x\sin\dfrac{1}{x}$

64) $\lim\limits_{x\to-1} \dfrac{3(x^2-1)}{\ln x^2}$

65) $\lim\limits_{x\to 0} \dfrac{\sin(5x)}{3x}$

66) $\lim\limits_{x\to\frac{\pi}{2}} 3\cdot(\sin x)^{\tan 2x}$

Evaluate each indefinite integral.

67) $\int (8x^3 - 15x^2 + 4)\, dx$

68) $\int \left(-25x^4 - \dfrac{24\sqrt[5]{x}}{5} + 8x^{-5}\right) dx$

69) $\int \left(\dfrac{10x^{\frac{3}{2}}}{2} + \dfrac{27x^{\frac{5}{4}}}{4} - 12x^{-5}\right) dx$

70) $\int -\dfrac{5}{\csc x}\, dx$

71) $\int 4\cot x \, dx$

72) $\int 9x^2(3x^3 - 1)^4 \, dx$

73) $\int (4x^5 - 1)^5 \cdot 80x^4 \, dx$

74) $\int \frac{(5 + \ln -x)^3}{x} \, dx$

75) $\int -\sec -x \cdot \tan -x \cdot \sec^5 -x \, dx$

76) $\int 12\csc -4x \cdot \cot -4x \cdot \csc^5 -4x \, dx$

77) $\int (e^x - 4)^3 \cdot e^x \, dx$

78) $\int \frac{25x^4}{\sqrt{1 - 25x^{10}}} \, dx$

79) $\int \frac{10x}{25 + 25x^4} \, dx$

Evaluate each definite integral.

80) $\int_{-2}^{1} (x^4 + x^3 - 2x^2 + 3)\, dx$

81) $\int_{-1}^{3} 4x^{\frac{1}{3}}\, dx$

82) $\int_{-3}^{-2} -\frac{3}{x^2}\, dx$

83) $\int_{-1}^{0} -2e^x\, dx$

84) $\int_{-3}^{-2} -\frac{1}{x}\, dx$

85) $\int_{-\frac{\pi}{6}}^{0} -2 \cdot \sec^2 x\, dx$

86) $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} -2\sin x\, dx$