Name Key

Block______ Date ______________________

Homework ______________________________

2.1 – 2.3 Proofs and Logic

Objectives:

A. I can give a proof of a conjecture.
B. I can draw a conclusion from a conditional.
C. I can state the converse of a conditional.
D. I can find counterexamples to disprove conditionals.
E. I can arrange statements into a logical chain and draw a conclusion.
F. I can write the definition of an object.
G. I can determine whether a statement is a definition.

Warm-UP!

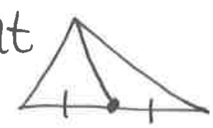
1. Write at least two sentences to convince me that you are a student at Lincoln Park.

2. Explain how you know that the following statement is not true:
All dice have six sides.

Define:

3. Coplanar – on the same plane

4. Median – a segment from a vertex to the midpoint of the opposite side



5. Inscribed Circle – a circle inside a triangle

6. Perpendicular - intersect at 90°

Let's talk about logic:

You can use logic to try to convince people that things are true when they are NOT.

Deductive reasoning: coming to a logical conclusion based on statements someone else gives to you

Counterexample: a Specific example that proves a claim false

Ex: Logical Arguments (Your goal is to convince someone that your claim is true.)

Claim: All houses are made of graham crackers

Reasoning: Graham Crackers are sturdy, they come in panels

Counterexample: My house is made of brick

(specific)

Logic with "If-Then" Statements:

Conditionals: "If-then" statements

They are represented as:

logical notation

If p, then q

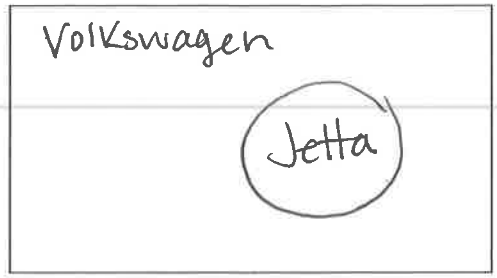
$p \Rightarrow q$ read "p implies q"

IF the doorbell rings, THEN Sabu barks

hypothesis — Conclusion

Ex: Conditionals

Euler Diagram



If a car is a Jetta, then it is a Volkswagen.

Hypothesis: A car is a Jetta

Conclusion: It is a Volkswagen

Reversing Conditionals:

Conditional: If I am the teacher, then I stand at the board during the lesson.

Hypothesis: I am the teacher

Conclusion: I stand at the board during the lesson

*** SWITCH ***

If I stand at the board during the lesson, then I am the teacher.

Converse: Switching the hypothesis & conclusion

The conditional is TRUE, but the converse is FALSE.

Counterexample: John (a student) stands at the board, but he is not the teacher.

Definitions or biconditionals:

Conditional: If a drink is made of coffee beans, then it is a coffee (beverage).

Converse: If a drink is a coffee (beverage), then it is made of coffee beans.

Are they both true? Yes

When the conditional and the converse are BOTH true, a biconditional or definition is formed.

$p \Leftrightarrow q$ read "p if and only if q" both mean the same thing

logical notation

The biconditional:

iff

A drink is made of coffee beans if and only if it is a coffee (beverage).

Practice:

Combine the two conditionals to form a biconditional:

If a street sign is red with eight sides, then the street sign is a stop sign.

If a street sign is a stop sign, then the street sign is red with eight sides.

A street sign is red with 8 sides iff the street sign is a stop sign.

Logical Chains: ** Logical Chain Activity**

Ex: Logical Chain:

Arrange the following statements in order to form a logical chain, then write the conditional statement that follows from the logic (summary conditional).

2 If I stub my toe, then I don't wear shoes.

4 If I get calluses, then I need a pedicure.

1 If I walk around in the dark, then I stub my toe.

3 If I don't wear shoes, then I get calluses.

Summary conditional:

If I walk around in the dark, then I need a pedicure.

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2.4 – 2.5 Properties, Theorems and Proofs

Objectives:

A. I can tell what a proof is.
B. I can use a two-column approach to a proof.
C. I can use an outline approach to a proof.
D. I can recognize algebraic and equivalence properties in proofs.
E. I can use algebraic and equivalence properties in proofs.

Proofs are logical arguments that something is true

We use two columns in writing proofs.

The first column shows work or steps.

The second shows reasoning or justification for those steps.

Prove: If 5x + 4 = 24, then x = 4

Work	Justification
$5x+4=24$	GIVEN (always the first line of a proof)
$5x=20$	Subtraction PoE
$x=4$	Division PoE

QED

quod erat demonstrandum

///

one of these at the end

Prove: If $3x - 2y = 16$ and $x = -4$, then $y = -14$

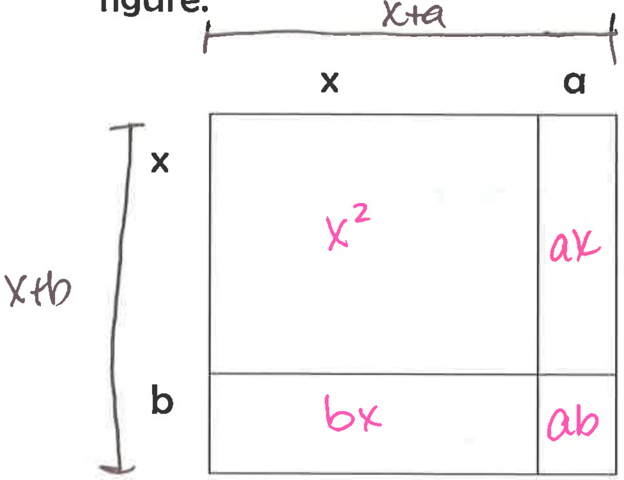
work	justification
$3x-2y=16$	given
$x=-4$	given
$3(-4)-2y=16$	substitution
$-12-2y=16$	multiplication
$-2y=28$	addition PoE
$y=-14$	division PoE ///

Reasoning

In some cases, reasoning may be conveyed is outline form to prove something as true.

This can be used in place of the two columns.

Prove that $(x + a)(x + b) = x^2 + ax + bx + ab$ given the following figure.



area = $l \cdot w$

whole sides — each piece added together

$(x+a)(x+b) = x^2+ax+bx+ab$

Indirect Proofs or Proofs by contradiction:

Assume the conclusion is the <u>exact opposite</u>, then come to a contradiction

Example of Proof by contradiction:

Given: $x(a+b) = 28, x = 4, and\ a\ is\ an\ integer$

Prove: $a \neq b$

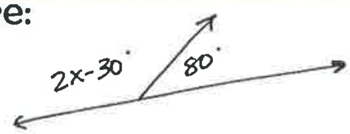
WORK	JUSTIFICATION
$x(a+b)=28$ $x=4$ a is an integer	Given
$a=b$	Assume for contradiction
$4(a+b)=28$	substitution
$4(a+a)=28$	substitution
$(a+a)=7$	division POE
$2a=7$	addition
$a=3.5$	division POE X L1
$\therefore a \neq b$	contradiction ///

Key

Special Angle Relationships:

Linear Pair:

Looks like:



How do I solve?

adds to 180°

Example:

$2x-30+80=180$ or $\begin{array}{r}180\\-80\\\hline100\end{array}$

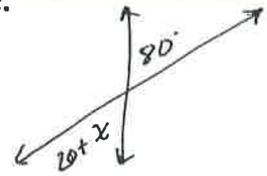
$2x+50=180$ $\quad$ $2x-30=100$

$2x=130$ $\quad$ $2x=130$

$x=65°$ $\quad$ $x=65°$

Vertical Angles:

Looks like:



How do I solve?

equal

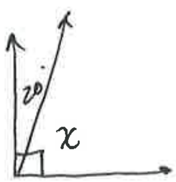
Example:

$20+x=80$

$x=60°$

Complementary Angles:

Looks like:



How do I solve?

adds to 90°

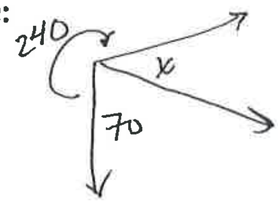
Example:

$20+x=90$

$x=70°$

"Circle" Angles:

Looks like:



How do I solve?

adds to 360°

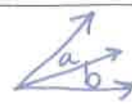
Example:

$240+70+x=360$

$310+x=360$

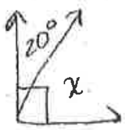
$x=50°$

adjacent - "next to" (only used if nothing better applies)



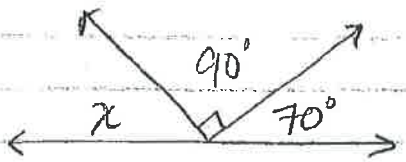
Key

Notes 2.4-2.5

1. 

90
-20
70° = X

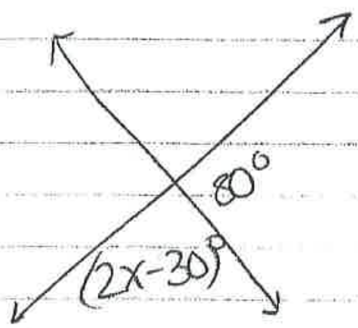
why? complimentary angles add to = 90°

2. 

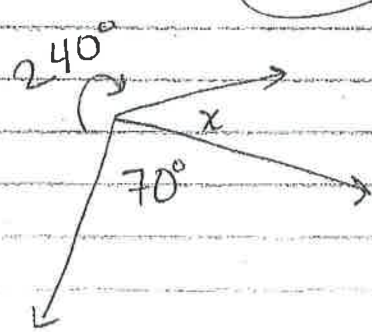
why? Right angles = 90°
Supplementary add to 180°

180
-90
90

90
-70
20° = X

3. 

180 why? Linear Pair adds to 180°
-80
100 = 2X-30 Addition Prop
+30 +30
130 = 2X Division Prop
$\frac{130}{2} = \frac{2X}{2}$
65° = X

4. 

circle adds to 360°

240
+70
310

360
-310
X = 50°

Proofs are convincing arguments that something is true

Different Methods of Proof

Justification Using Properties of Equality and Congruence

PROPERTIES OF EQUALITY FOR REAL NUMBERS	
Reflexive Property	For any number a, $a = a$.
Symmetric Property	For any numbers a and b, if $a = b$ then $b = a$.
Transitive Property	For any numbers a, b and c, if $a = b$ and $b = c$, then $a = c$.
Addition and Subtraction Properties	For any numbers a, b and c, if $a = b$, then $a + c = b + c$ and $a - c = b - c$.
Multiplication and Division Properties	For any numbers a, b and c, if $a = b$, then $a \times c = b \times c$ and if $c \neq 0$, then $a \div c = b \div c$.
Substitution Property	For any numbers a and b, if $a = b$, then a may be replaced with b in any equation.

PROPERTIES OF CONGRUENCE	
Reflexive Property of Congruence	$\overline{AB} \cong \overline{AB}$
Symmetric Property of Congruence	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$
Transitive Property of Congruence	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$