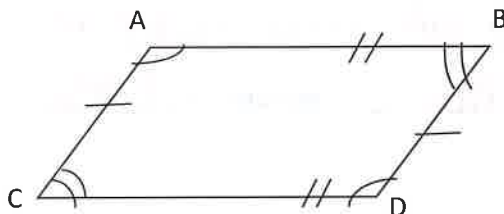


4.1 - 4.2 Congruent Polygons & Triangles

Objectives:

- A. I can properly name congruent polygons.
- B. I can find congruent angles or sides either from a picture of congruent polygons or from their congruence statement.
- C. I can prove that triangles are congruent using the triangle congruence postulates of AAS, SSS, SAS, ASA.

Warm-UP!



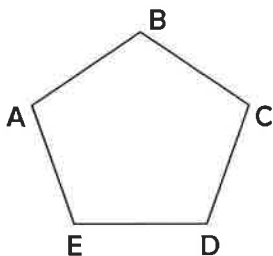
1. Properly name the congruent sides of the figure.

2. Properly name the congruent angles of the figure.

-----STOP-----

Naming Polygons: There are multiple correct ways to name a polygon.

Name the pentagon:



1. pentagon ABCDE
2. BCDEA
3. CDEAB
4. DEABC
5. EABCD

Clockwise!

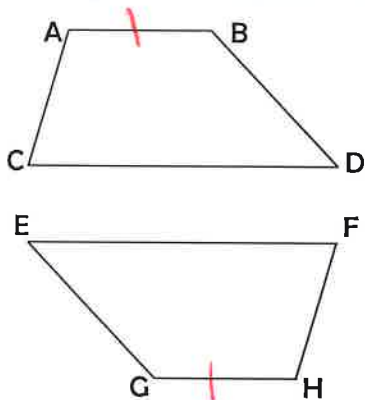
Congruent Polygons

If two polygons are congruent, then they have congruent sides and congruent angles.

When naming congruent polygons, vertices must be named in the same order so that they correspond.

What does that mean? *The sides (or angles) that match need to be named in the same order for both polygons.*

Name the congruent quadrilaterals:



- quad* ABDC is congruent to *quad* HGEF
2. BDCA is congruent to GEFH
3. DCAB is congruent to EFHG
4. CABD is congruent to FHGE

Using CPCTC

Say what? *Corresponding Parts of Congruent Triangles are Congruent*

Since congruent polygons are named by their corresponding parts, we don't need pictures in order to determine congruent angles and sides.

Congruent parts can be named based on their location in the figures' names.

Name the congruent parts if quadrilateral $ABCD \cong$ quadrilateral $WXYZ$:

Angles:

- $\angle A \cong \angle W$
- $\angle B \cong \angle X$
- $\angle C \cong \angle Y$
- $\angle D \cong \angle Z$

Sides:

- $\overline{AB} \cong \overline{WX}$
- $\overline{BC} \cong \overline{XY}$
- $\overline{CD} \cong \overline{YZ}$
- $\overline{DA} \cong \overline{ZW}$

4.2 TRIANGLE CONGRUENCE

sides = pink
angles = green

Side-Side-Side Postulate: SSS

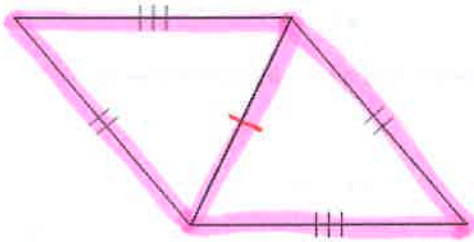


If each side of a triangle is congruent to a side of another triangle, then the two triangles are congruent.

Sketch:



Example:



Side-Angle-Side Postulate: SAS

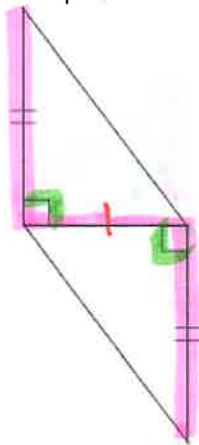


If two consecutive sides and their *included* angle in a triangle are congruent to two consecutive sides and their *included* angle in another triangle, then the two triangles are congruent.

Sketch:



Example:



Angle-Side-Angle Postulate: ASA

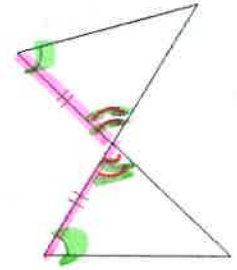


If two consecutive angles and their *included* side in a triangle are congruent to two consecutive angles and their *included* side in another triangle, then the two triangles are congruent.

Sketch:



Example:



Angle-Angle-Side Postulate: AAS

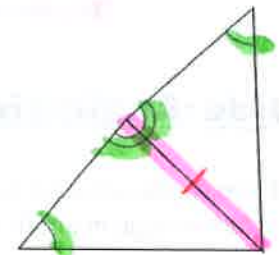


If two angles and a *non-included* side of a triangle are congruent to two angles and a *non-included* side of another triangle, then the two triangles are congruent.

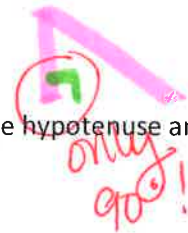
Sketch:



Example:



Hypotenuse-Leg Postulate: HL

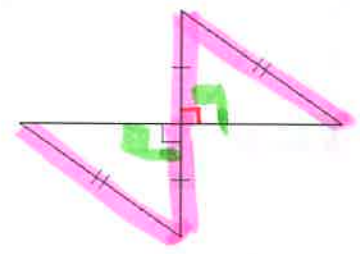


If the hypotenuse and leg of a right triangle are congruent to the hypotenuse and leg of another right triangle, then the two triangles are congruent.

Sketch:



Example:



Yes
SSS
SAS
ASA
AAS
HL

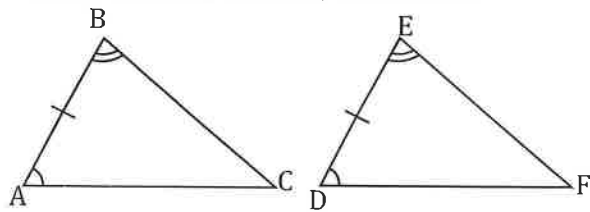
No
SSA
AAA
trickery
not enough info

43 Proofs Involving Congruent Triangles Key

For these fill in any missing statements or reasons.

1.

Given: $\overline{AB} \cong \overline{DE}$, $\angle B \cong \angle E$, and $\angle A \cong \angle D$

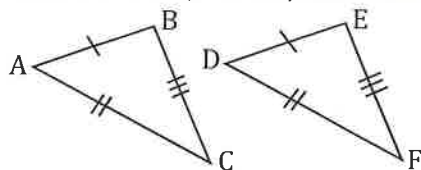


Prove: $\triangle ABC \cong \triangle DEF$

Statements	Reasons
1. $\overline{AB} \cong \overline{DE}$	1. Given
2. $\angle B \cong \angle E$	2. Given
3. $\angle A \cong \angle D$	3. Given
4. $\triangle ABC \cong \triangle DEF$	4. ASA ///

3.

Given: $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, and $\overline{BC} \cong \overline{EF}$

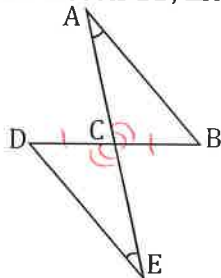


Prove: $\triangle ABC \cong \triangle DEF$

Statements	Reasons
1. $\overline{AB} \cong \overline{DE}$	1. Given
2. $\overline{AC} \cong \overline{DF}$	2. Given
3. $\overline{BC} \cong \overline{EF}$	3. Given
4. $\triangle ABC \cong \triangle DEF$	4. SSS ///

5.

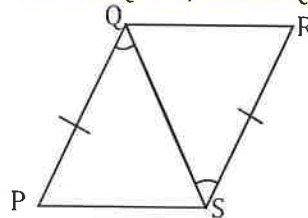
Given: \overline{AE} bisects \overline{BD} , $\angle A \cong \angle E$



Prove: $\triangle ABC \cong \triangle EDC$

Statements	Reasons
1. $\angle A \cong \angle E$	1. Given
2. \overline{AE} bisects \overline{BD}	2. Given
3. $\overline{DC} \cong \overline{BC}$	3. Definition of Bisect
4. $\angle ACB \cong \angle DCE$	4. vertical angles
5. $\triangle ABC \cong \triangle EDC$	5. AAS ///

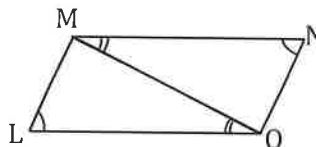
2. Given: $\overline{PQ} \cong \overline{RS}$, and $\angle PQS \cong \angle RSQ$



Prove: $\triangle PQS \cong \triangle RSQ$

Statements	Reasons
1. $\angle PQS \cong \angle RSQ$	1. Given
2. $\overline{PQ} \cong \overline{RS}$	2. Given
3. $\overline{QS} \cong \overline{QS}$	3. Reflexive Property
4. $\triangle PQS \cong \triangle RSQ$	4. SAS ///

4. Given: $\angle L \cong \angle N$, $\angle LOM \cong \angle NMO$ ← Cross out

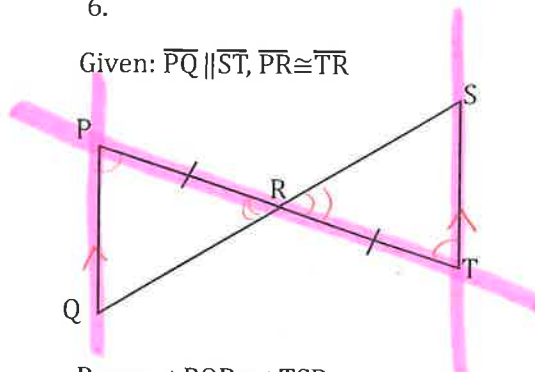


Prove: $\triangle LMO \cong \triangle NMO$

Statements	Reasons
1. $\angle L \cong \angle N$	1. Given
2. $\angle LOM \cong \angle NMO$	2. Given
3. $\overline{MO} \cong \overline{MO}$	3. Reflexive Property
4. $\triangle LMO \cong \triangle NMO$	4. AAS ///

6.

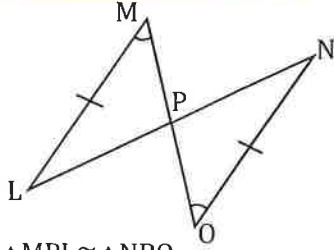
Given: $\overline{PQ} \parallel \overline{ST}$, $\overline{PR} \cong \overline{TR}$



Prove: $\triangle PQR \cong \triangle TSR$

Statements	Reasons
1. $\overline{PR} \cong \overline{TR}$	1. Given
2. $\overline{PQ} \parallel \overline{ST}$	2. Given
3. $\angle P \cong \angle T$	3. Alternate Interior Angles
4. $\angle ACB \cong \angle DCE$	4. vertical angles
5. $\triangle PQR \cong \triangle TSR$	5. ASA ///

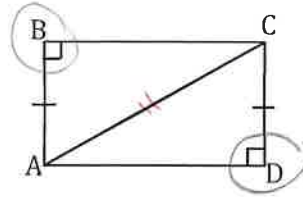
7. Given: $\overline{LM} \cong \overline{NO}$, and $\angle M \cong \angle O$



Prove: $\triangle MPL \cong \triangle NPO$

Statements	Reasons
1. $\overline{LM} \cong \overline{NO}$	1. Given
2. $\angle M \cong \angle O$	2. Given
3. $\angle MPL \cong \angle NPO$	3. vertical angles
4. $\triangle MPL \cong \triangle NPO$	4. AAS ///

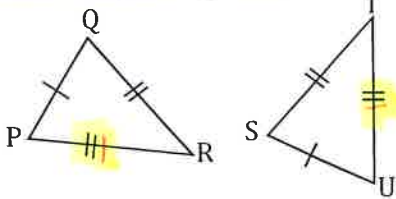
8. Given: $\overline{AB} \cong \overline{DC}$



Prove: $\triangle ABC \cong \triangle CDA$

Statements	Reasons
1. $\overline{AB} \cong \overline{DC}$	1. Given
2. $\overline{AC} \cong \overline{AC}$	2. Reflexive Property
3. $\triangle ABC \cong \triangle CDA$	3. HL ///

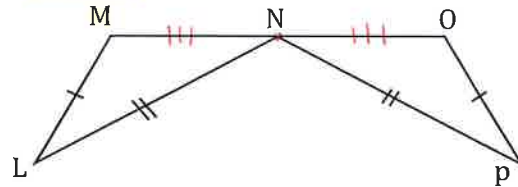
9. Given: $\overline{PQ} \cong \overline{SU}$, $\overline{QR} \cong \overline{ST}$, and $\overline{PR} \cong \overline{TU}$



Prove: $\triangle PQR \cong \triangle STU$

Statements	Reasons
1. $\overline{PQ} \cong \overline{SU}$	1. Given
2. $\overline{QR} \cong \overline{ST}$	2. Given
3. $\overline{PR} \cong \overline{TU}$	3. Given
4. $\triangle PQR \cong \triangle STU$	4. SSS ///

10. Given: N is the midpoint of \overline{MO} , $\overline{LM} \cong \overline{OP}$, and $\overline{LN} \cong \overline{PN}$

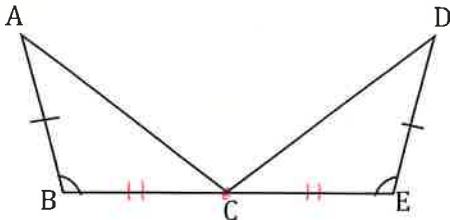


Prove: $\triangle LMN \cong \triangle PON$

Statements	Reasons
1. $\overline{LM} \cong \overline{OP}$	1. Given
2. $\overline{LN} \cong \overline{PN}$	2. Given
3. N is the Midpoint of \overline{MO}	3. Given
4. $\overline{MN} \cong \overline{NO}$	4. Midpoint Definition
5. $\triangle LMN \cong \triangle PON$	5. SSS ///

11.

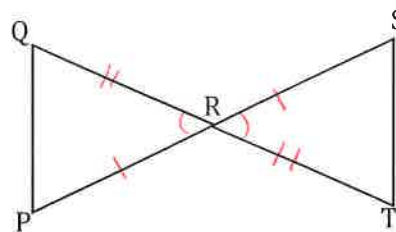
- Given: C is the midpoint of \overline{BE} , $\angle B \cong \angle E$, and $\overline{AB} \cong \overline{DE}$



Prove: $\triangle ABC \cong \triangle DEC$

Statements	Reasons
1. $\angle B \cong \angle E$	1. Given
2. $\overline{AB} \cong \overline{DE}$	2. Given
3. C is the midpoint of \overline{BE}	3. Given
4. $\overline{BC} \cong \overline{CE}$	4. Midpoint Definition
5. $\triangle ABC \cong \triangle DEC$	5. SAS ///

12. Given: \overline{QT} bisects \overline{SP} , \overline{SP} bisects \overline{QT}



Prove: $\triangle QRP \cong \triangle SRT$

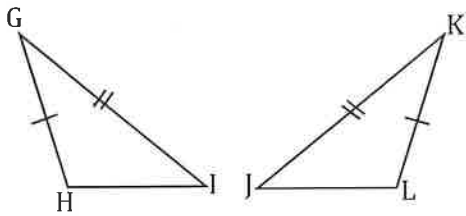
Statements	Reasons
1. \overline{QT} bisects \overline{SP}	1. Given
2. \overline{SP} bisects \overline{QT}	2. Given
3. $\overline{QR} \cong \overline{TR}$	3. Definition of Bisect
4. $\overline{PR} \cong \overline{SR}$	4. Definition of Bisect
5. $\angle QRP \cong \angle SRT$	5. Vertical Angles
6. $\triangle QRP \cong \triangle SRT$	6. SAS ///

4.4 Proofs involving CPCTC

Key

Fill in the missing information in each proof.

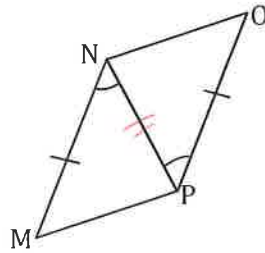
4. Given: $\overline{GH} \cong \overline{KL}$, $\angle G \cong \angle K$, and $\overline{GI} \cong \overline{KI}$



Prove: $\overline{HI} \cong \overline{LI}$

Statements	Reasons
1. $\overline{GH} \cong \overline{KL}$	1. Given
2. $\angle G \cong \angle K$	2. Given
3. $\overline{GI} \cong \overline{KI}$	3. Given
4. $\triangle GHI \cong \triangle KLI$	4. SAS
5. $\overline{HI} \cong \overline{LI}$	5. CPCTC ///

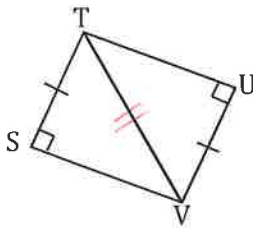
5. Given: $\angle MNP \cong \angle OPN$, and $\overline{MN} \cong \overline{OP}$



Prove: $\overline{MP} \cong \overline{NO}$

Statements	Reasons
1. $\angle MNP \cong \angle OPN$	1. Given
2. $\overline{MN} \cong \overline{OP}$	2. Given
3. $\overline{NP} \cong \overline{NP}$	3. Reflexive
4. $\triangle MNP \cong \triangle OPN$	4. SAS
5. $\overline{MP} \cong \overline{NO}$	5. CPCTC ///

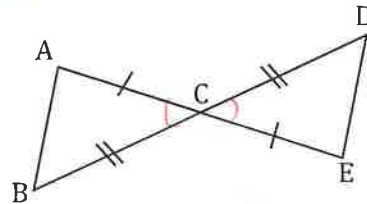
6. Given: $\overline{ST} \cong \overline{VU}$



Prove: $\angle SVT \cong \angle UTU$

Statements	Reasons
1. $\overline{ST} \cong \overline{VU}$	1. Given
2. $\overline{TV} \cong \overline{TV}$	2. Reflexive Property
3. $\triangle TVS \cong \triangle TVU$	3. HL
4. $\angle SVT \cong \angle UTU$	4. CPCTC ///

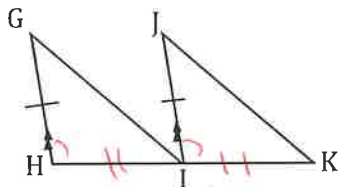
7. Given: $\overline{AC} \cong \overline{CE}$, $\overline{DC} \cong \overline{BC}$



Prove: $\angle B \cong \angle D$

Statements	Reasons
1. $\overline{AC} \cong \overline{CE}$	1. Given
2. $\overline{BC} \cong \overline{CD}$	2. Given
3. $\angle ACB \cong \angle DCE$	3. vertical angles
4. $\triangle ABC \cong \triangle DEC$	4. SAS
5. $\angle B \cong \angle D$	5. CPCTC ///

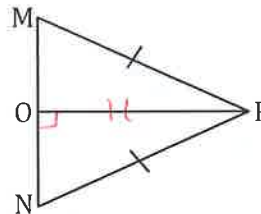
8. Given: $\overline{GH} \parallel \overline{JI}$, I is the midpoint of \overline{HK} and $\overline{GH} \cong \overline{JI}$



Prove: $\angle G \cong \angle J$

Statements	Reasons
1. $\overline{GH} \parallel \overline{JI}$	1. Given
2. I is the midpoint of \overline{HK}	2. Given
3. $\overline{GH} \cong \overline{JI}$	3. Given
4. $\overline{HI} \cong \overline{IK}$	4. Midpoint Definition
5. $\angle GHI \cong \angle JIK$	5. Corresponding
6. $\triangle GHI \cong \triangle JIK$	6. SAS
7. $\angle G \cong \angle J$	7. CPCTC ///

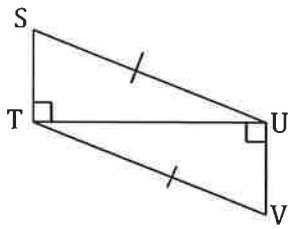
9. Given: $\overline{MP} \cong \overline{NP}$, $\overline{MN} \perp \overline{OP}$



Prove: $\overline{MO} \cong \overline{ON}$

Statements	Reasons
1. $\overline{MP} \cong \overline{NP}$	1. Given
2. $\overline{MN} \perp \overline{OP}$	2. Given
3. $\overline{OP} \cong \overline{OP}$	3. Reflexive
4. $\triangle MOP \cong \triangle NOP$	4. HL
5. $\overline{MO} \cong \overline{ON}$	5. CPCTC ///

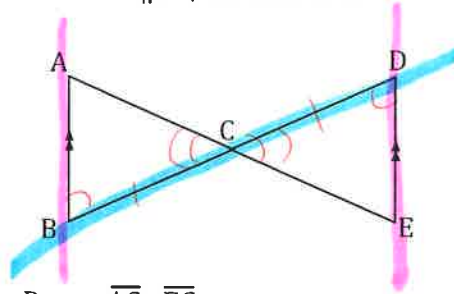
10. Given: $\overline{SU} \cong \overline{VT}$



Prove: $\overline{ST} \cong \overline{UV}$

Statements	Reasons
1. $\overline{SU} \cong \overline{VT}$	1. Given
2. $\overline{TU} \cong \overline{TU}$	2. Reflexive
3. $\triangle STU \cong \triangle VUT$	3. HL
4. $\overline{ST} \cong \overline{UV}$	4. CPCTC ///

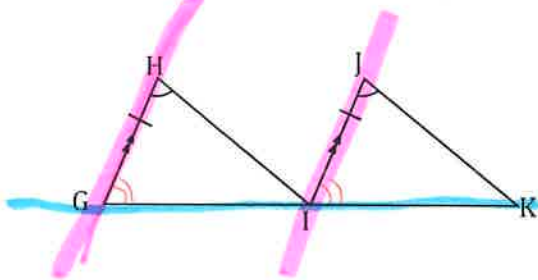
11. Given: $\overline{AB} \parallel \overline{DE}$, \overline{AE} bisects \overline{BD}



Prove: $\overline{AC} \cong \overline{EC}$

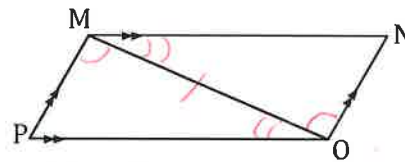
Statements	Reasons
1. $\overline{AB} \parallel \overline{DE}$	1. Given
2. \overline{AE} bisects \overline{BD}	2. Given
3. $\angle ABC \cong \angle EDC$	3. Alternate Interior
4. $\angle ACB \cong \angle DCE$	4. Vertical angles
5. $\overline{BC} \cong \overline{CD}$	5. Def of Bisect
6. $\triangle ABC \cong \triangle EDC$	6. ASA
7. $\overline{AC} \cong \overline{EC}$	7. CPCTC ///

12. Given: $\overline{GH} \parallel \overline{IJ}$, $\angle H \cong \angle J$ and $\overline{GH} \cong \overline{IJ}$



Prove: $\angle GIH \cong \angle IKJ$

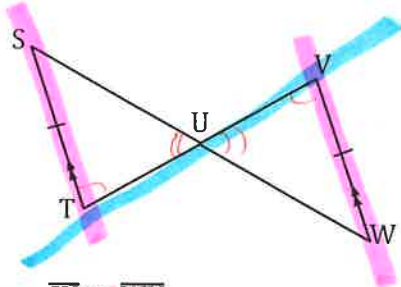
Statements	Reasons
1. $\overline{GH} \parallel \overline{IJ}$	1. Given
2. $\angle H \cong \angle J$	2. Given
3. $\overline{GH} \cong \overline{IJ}$	3. Given corresponding
4. $\angle G \cong \angle JIK$	4. Alternate Interior
5. $\triangle GHK \cong \triangle IJK$	5. ASA
6. $\angle GIH \cong \angle IKJ$	6. CPCTC ///



Prove: $\overline{PM} \cong \overline{ON}$

Statements	Reasons
1. $\overline{PM} \parallel \overline{ON}$	1. Given
2. $\overline{PN} \parallel \overline{PO}$	2. Given
3. $\angle PMO \cong \angle NOP$	3. Alternate Interior
4. $\angle MOP \cong \angle NMO$	4. Alternate Interior
5. $\overline{MO} \cong \overline{MO}$	5. Reflexive
6. $\triangle MOP \cong \triangle ONM$	6. ASA
7. $\overline{PM} \cong \overline{ON}$	7. CPCTC ///

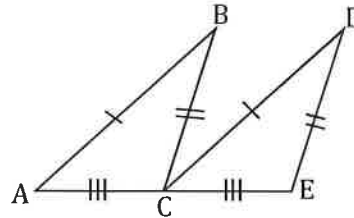
14. Given: $\overline{ST} \parallel \overline{WV}$, and $\overline{ST} \cong \overline{WV}$



Prove: $\overline{SU} \cong \overline{WU}$

Statements	Reasons
1. $\overline{ST} \parallel \overline{WV}$	1. Given
2. $\overline{ST} \cong \overline{WV}$	2. Given
3. $\angle STU \cong \angle WVU$	3. Alternate Interior
4. $\angle SUT \cong \angle WUV$	4. Vertical
5. $\triangle STU \cong \triangle WVU$	5. AAS
6. $\overline{SU} \cong \overline{WU}$	6. CPCTC ///

15. Given: $\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{DE}$, and $\overline{AC} \cong \overline{CE}$



Prove: $\angle A \cong \angle DCE$

Statements	Reasons
1. $\overline{AB} \cong \overline{CD}$	1. Given
2. $\overline{BC} \cong \overline{DE}$	2. Given
3. $\overline{AC} \cong \overline{CE}$	3. Given
4. $\triangle ABC \cong \triangle CDE$	4. SSS
5. $\angle A \cong \angle DCE$	5. CPCTC ///

4.3 Midsegments

Objectives:

A.1 can find missing segments in triangles and trapezoids with midsegments.

Midsegments in Triangles

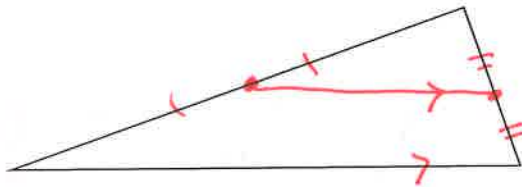
The midsegment of a triangle:

Connects the midpoint of two sides

Is parallel to the third side.

Is half the length of the third side.

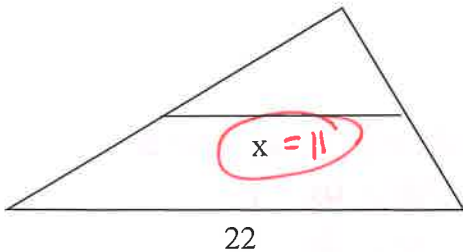
Sketch:



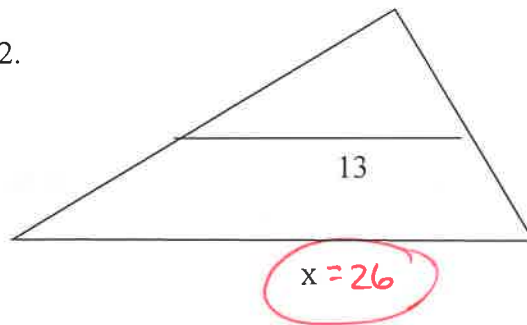
2 · midsegment = base

Finding the missing values:

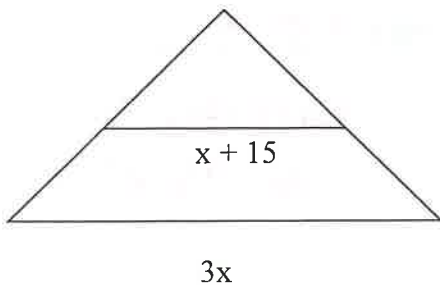
1.



2.



3.



Solve for x:

$$\begin{aligned}
 2(x+15) &= 3x \\
 2x+30 &= 3x \\
 \underline{-2x} \quad \underline{-2x} & \\
 30 &= x
 \end{aligned}$$

Midsegment length: $x+15 = 30+15 = 45$

Base length: $3x = 3(30) = 90$ $2(45) = 90$ ✓

Midsegments in Trapezoids

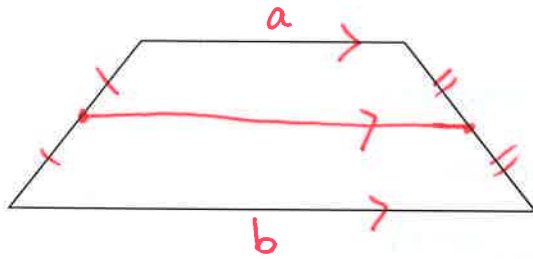
The midsegment of a trapezoid:

Connects the midpoints of the two non-parallel sides.

Is parallel to the two bases.

Is equal to half the sum of the two bases.

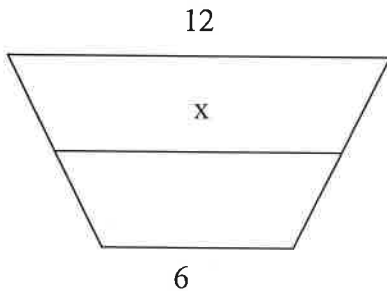
Sketch:



$$2(\text{midsegment}) = a + b$$

Finding the missing values:

1.

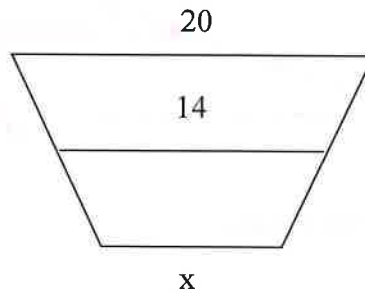


$$2x = 12 + 6$$

$$2x = 18$$

$$x = 9$$

2.

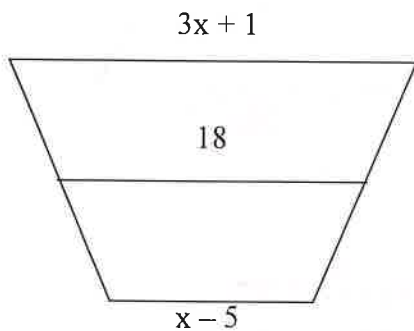


$$2(14) = 20 + x$$

$$28 = 20 + x$$

$$8 = x$$

3.



Solve for x:

$$2(18) = 3x + 1 + x - 5$$

$$36 = 4x - 4$$

$$40 = 4x$$

$$10 = x$$

$3x + 1$ Base length: $3(10) + 1 = 31$

$x - 5$ Base length: $10 - 5 = 5$

$$2(18) = 31 + 5$$

$$36 = 36 \checkmark$$

4.5 Special Quadrilaterals

Objectives:

- A. I can classify quadrilaterals by their type.
- B. I can identify and mark the congruent or parallel sides of quadrilaterals.
- C. I can identify and mark the congruent angles of quadrilaterals

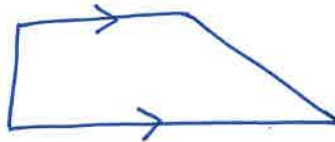
Quadrilaterals are: 4-sided polygons

Trapezoid

Properties:

1. Has one and only one pair of parallel sides

Sketches:



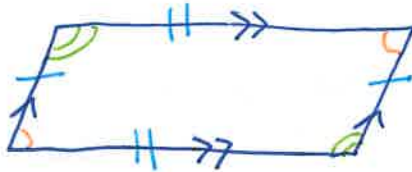
Parallelogram

Properties:

1. Has exactly two pairs of parallel sides
2. These pairs are also congruent
3. Consecutive angles are supplementary
4. Opposite angles are congruent

$$\text{orange} + \text{green} = 180^\circ$$

Sketches:



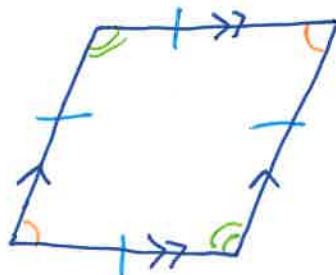
Rhombus

Properties:

1. Has exactly two pairs of parallel sides
2. All four sides are congruent
3. Consecutive angles are supplementary
4. Opposite angles are congruent

$$\text{orange} + \text{green} = 180^\circ$$

Sketches:

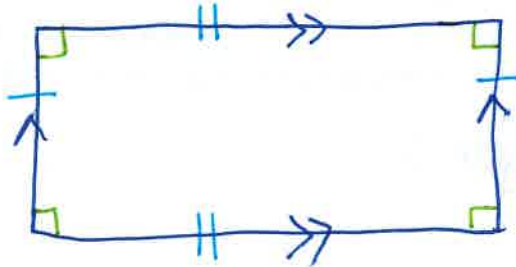


Rectangle

Properties:

1. Has exactly two pairs of parallel sides
2. These pairs are also congruent
3. Any two angles are supplementary
4. All four angles are congruent

Sketches:

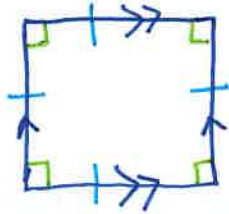


Square

Properties:

1. Has exactly two pairs of parallel sides
2. All four sides are congruent
3. Any two angles are supplementary
4. All four angles are congruent

Sketches:



Quadrilaterals that overlap

Trapezoids and parallelograms are quadrilaterals.

Euler Diagram:

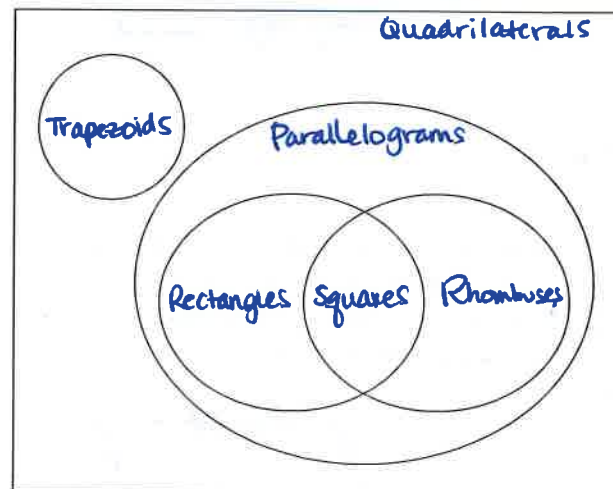
A rhombus is always a: parallelogram

A rectangle is always a: parallelogram

A square is always a: parallelogram

rhombus

rectangle



4.6 – Diagonals in Quadrilaterals

Objectives:

A.I can apply the properties of parallel lines and transversals to the diagonals of a quadrilateral.

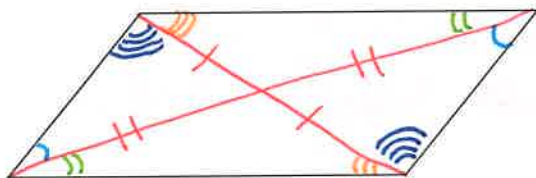
B.I can solve quadrilateral problems with diagonals in them.

Parallelogram:

Properties:

- Opposite sides are congruent
- Opposite sides are parallel
- Opposite angles are congruent
- Consecutive angles are supplementary

In the figure below, sketch both diagonals of the parallelogram.



Examine the diagonals of the parallelogram. What do you see?

Special Property: *They bisect each other*

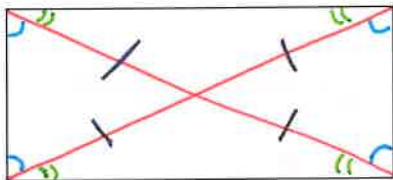
Rectangle

Properties of a rectangle:

- Opposite sides are congruent
- Opposite sides are parallel
- All four angles are congruent
- Consecutive angles are supplementary

blue + green = 90°

In the figure below, sketch both diagonals of the rectangle.



** Note: There are two pairs of isosceles triangles.*

Examine the diagonals of the rectangle. What do you see?

Like a parallelogram: *They bisect each other*

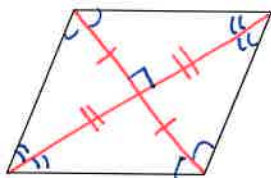
Special Property: *The diagonals are congruent*

Rhombus

Properties of a rhombus:

- All four sides are congruent
- Opposite sides are parallel
- Opposite angles are congruent
- Consecutive angles are supplementary

In the figure below, sketch both diagonals of the rhombus.



Examine the diagonals of the rhombus. What do you see?

Like a parallelogram: *The diagonals bisect each other.*

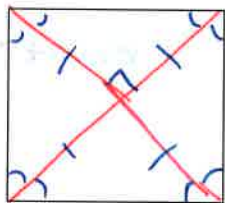
Special Property: *The diagonals are perpendicular.
The diagonals are angle bisectors.*

Square

Properties of a square:

- All four sides are congruent
- Opposite sides are parallel
- All four angles are congruent
- Consecutive angles are supplementary

In the figure below, sketch both diagonals of the square.



Examine the diagonals of the square. What do you see?

Like a parallelogram: *The diagonals bisect each other.*

Like a rectangle: *The diagonals are equal.*

Like a rhombus: *The diagonals are perpendicular.
The diagonals are angle bisectors.*