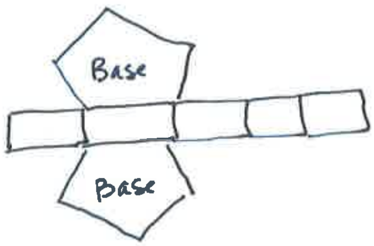
8.0 Nets

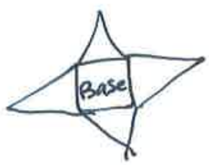
Nets - the two-dimensional cutout for a 3D object

Prisms - 2 identical bases, all other pieces are rectangles

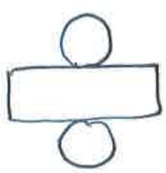


Pentagonal Prism

Pyramid - 1 base, all other pieces triangles



Cylinder - 2 circles, one rectangle



Cone - 1 circle, one "cape"

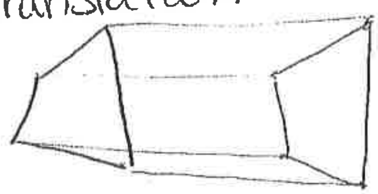
no net for a sphere

8.1 & 8.2 Notes

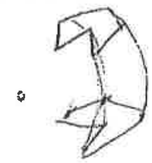
8.1 Dilations and Scale Factors

- Recall: a few units ago we did transformations:
  - translation, rotation, reflections

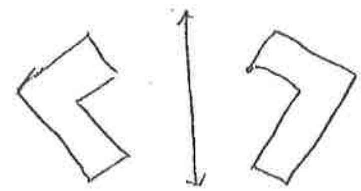
• translation



• rotation



• reflection

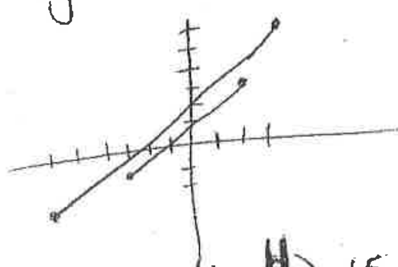


- • changes position, but not size & shape, so they are congruent figures

- Dilations (in general)
- Dilations on the coordinate plane
  - • change size, but keep shape
  - • <u>not</u> congruent
  - • ~~on coordinate plane~~: • n - scale factor
    - ★ multiply everything by scale factor

Ex: segment (2,3) & (-3,-1)

$n=2$

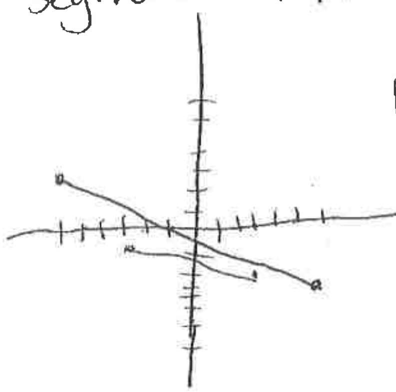


new points (4,6) & (-6,-2)

Ex: segment (6,-4) (-6,2)

$n=\frac{1}{2}$

★ multiply everything by scale factor



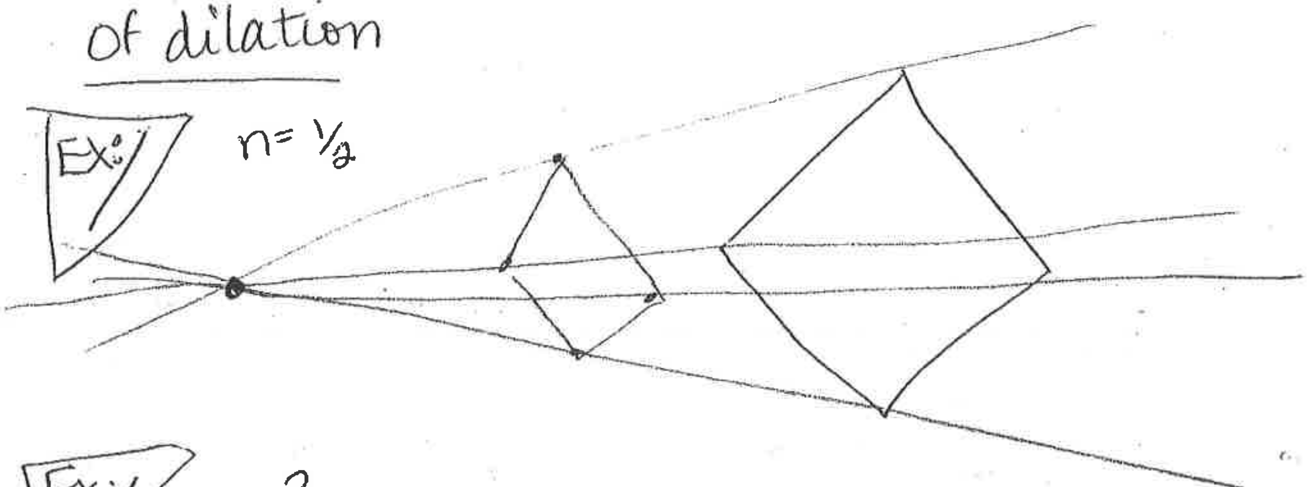
new points

(3,-2)(-3,1)

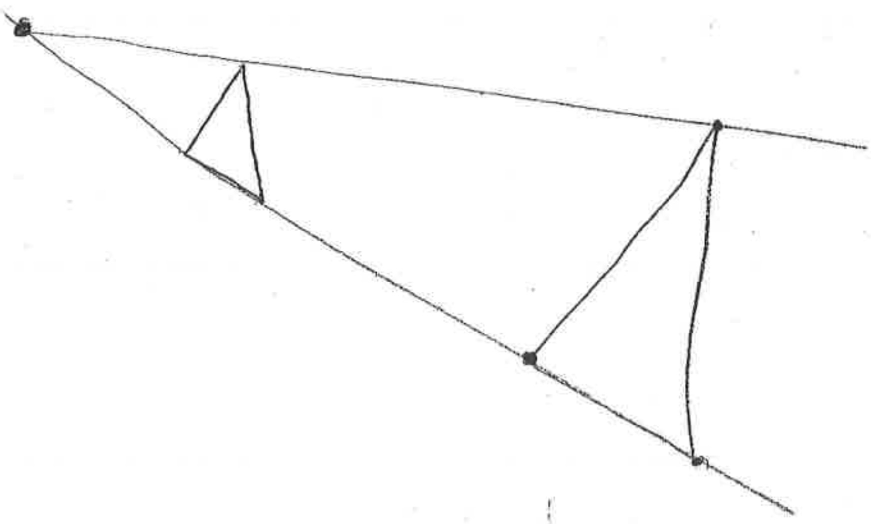
- if the figure gets smaller, it's a contraction
- If the figure gets larger, it's an expansion.

- Dilations without a coordinate plane
  - figures contract or expand ~~around~~ using a center of dilation

Ex: $n = \frac{1}{2}$



Ex: $n = 3$



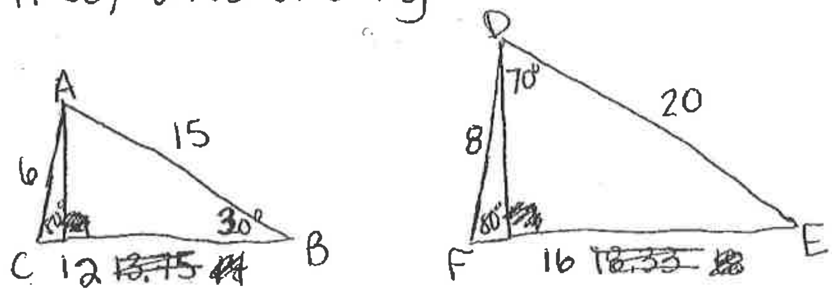
8.2 Similar Polygons

- Similarity
  - Two figures are similar iff one is congruent to the image of the other by dilation

aka:
- corresponding angles are <u>equal</u>
- corresponding sides are <u>proportional</u>

Ex: 1

Determine if the two triangles are similar - if so, write a congruence statement.



- all angles are equal
- ratios $\frac{15}{20} = \frac{3}{4}$ $\frac{6}{8} = \frac{3}{4}$ $\frac{12}{16} = \frac{3}{4}$ all proportional
- so $\triangle ABC \sim \triangle DEF$

↑ similar

~~remembent~~ remember $\cong$ means congruent

$\frac{2}{5}$

$\frac{4}{3} = \frac{14}{x}$

$42 = 4x$

$10.5 = x$

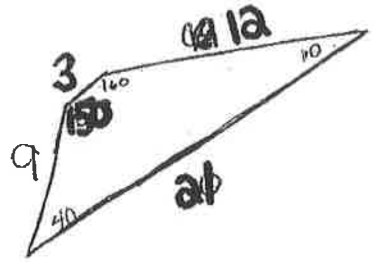
70
30
150
250
110

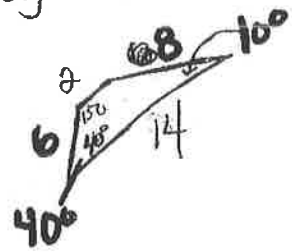
Ex: 2

Assume ~~the polygons are similar.~~

Fill in every missing value.

$n = \frac{3}{2}$





360

10

40

310

160

150

$\frac{12}{8} = \frac{3}{2}$ $\frac{21}{14} = \frac{3}{2}$ $\frac{9}{6} = \frac{3}{2}$ $\frac{3}{2} = \frac{3}{2}$

15

120

95

230

130

$\frac{3}{2} = \frac{x}{9}$

$27 = 2x$

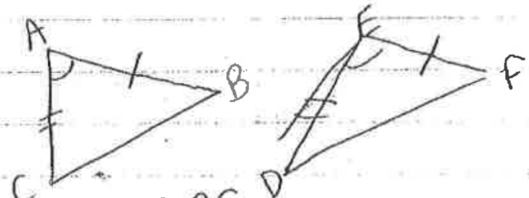
$13.5 = x$

$\frac{4}{9} = \frac{16}{36} =$

8.3 Triangle Similarity

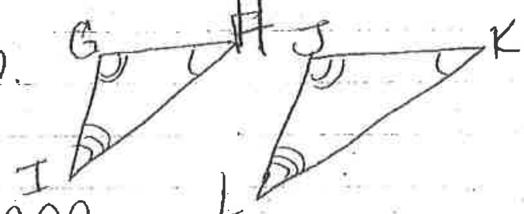
Warm-up: Determine if the two triangles are congruent. If so, write the congruence statement.
Recall congruence ~~theorems~~ Postulates: SSS, SAS, ASA, AAS, ~~SSA~~, HL, ~~AAA~~

1.



yes, SAS
$\triangle CAB \cong \triangle DEF$

2.



AAA, no
not congruent

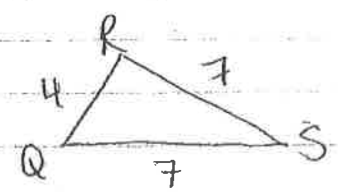
Proving similar triangles

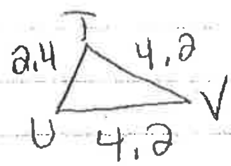
- If triangles are congruent, they are similar. (all above postulates prove similarity).

- Since similarity deals with all equal angles: AAA or AA shows similar triangles.
  Ex: 1 (warm-up #2) shows similar triangles
  $\triangle HGI \sim \triangle KJL$ ☆ order matters

- Since similarity deals with proportional sides: SSS show similarity if sides are proportional

Ex: 2





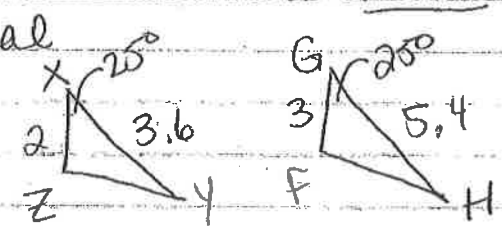
$\triangle RSQ \sim \triangle TVU$

$\frac{RS}{TV} = \frac{7}{4.2} = \frac{5}{3}$ $\frac{SQ}{VU} = \frac{7}{4.2} = \frac{5}{3}$ $\frac{QR}{UT} = \frac{4}{2.4} = \frac{5}{3}$

• Since similarity deals with equal angles and proportional sides:

SAS shows similarity if two sides are proportional and their <u>included</u> angle are equal

Ex:3



$\angle X = \angle G$ (included $\angle$)

$\frac{YX}{HG} = \frac{3.6}{5.4}$ $\frac{2}{3}$ $\quad \frac{XZ}{GF} = \frac{2}{3}$ $\quad \Delta ZXY \sim \Delta FGH$

• Why are ASA and AAS not included in similarity?

- b/c both have <u>AA</u> two angles, which prove similarity

Hw: Pg. 521 all

8.4 A- MIDSEGMENTS IN TRIANGLES AND TRAPEZOIDS

Midsegment in Triangles

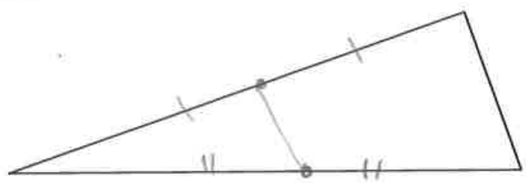
The midsegment of a triangle:

Connects the midpoints of two sides

Is parallel to the third side.

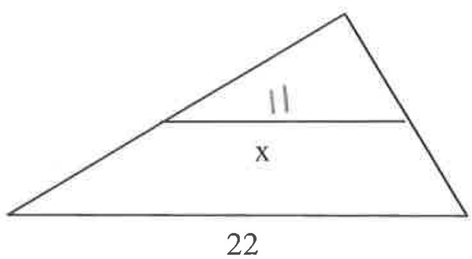
Is $\frac{1}{2}$ of the third side.

Sketch:

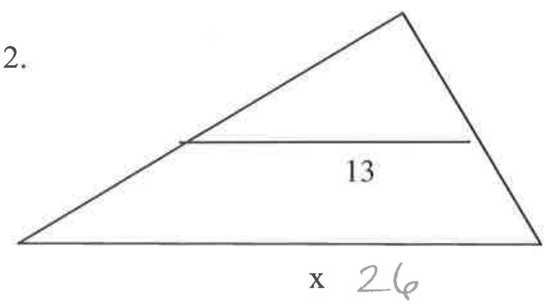


Finding the missing values:

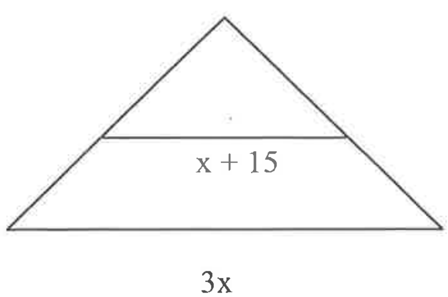
1.



2.



3.



Solve for x:

$3x = 2(x+15)$

$3x = 2x + 30$

$-2x \quad -2x$

$x = 30$

Midsegment length: 45

Base length: 90

Midsegments in Trapezoids

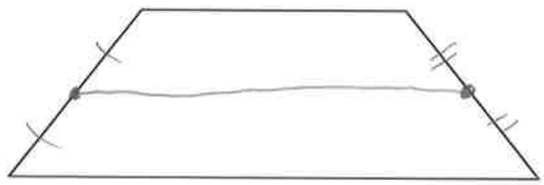
The midsegment of a trapezoid:

Connects the midpoints of the two nonparallel sides.

Is parallel to the two parallel.

Is equal to $\frac{1}{2}(b_1+b_2)$ of the two bases.

Sketch:



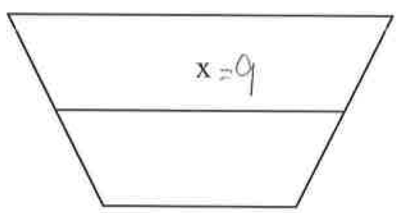
Finding the missing values:

$\frac{1}{2}(20+x)=14$

$20+x=28$

1.

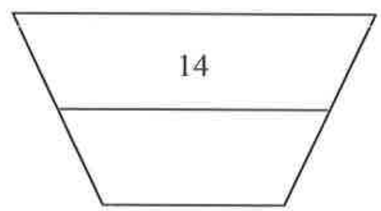
12



6

2.

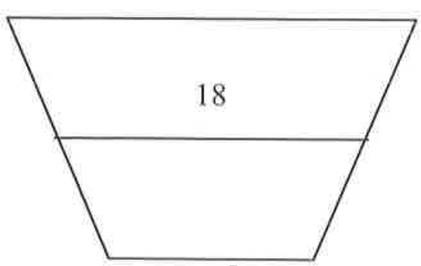
20



x = 8

3.

3x + 1



x – 5

$\frac{1}{2}[(3x+1)+(x-5)]=18$

Solve for x: $3x+1+x-5=36$

$4x-4=36$

$4x=40$

$x=10$

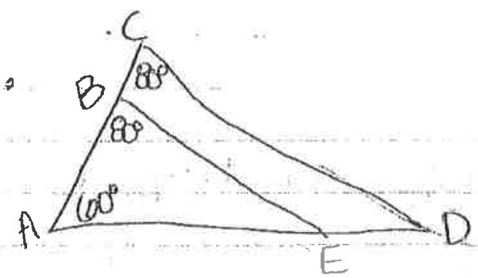
3x + 1 Base length: 31

x – 5 Base length: 5

8.4 The Side-Splitting Theorem

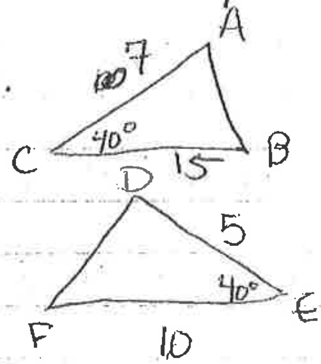
Warm-up: Determine whether the triangles are similar, if so, write a similarity statement.

1.



$\triangle ABE \sim \triangle ACD$
by AA

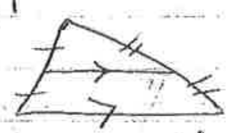
2.



no, sides are not proportional

The Side-Splitting Theorem

- Recall midsegment:



- If the segment in the triangle that is parallel does not cut the sides in ½, it cuts it proportionally.

Ex: 1)

you can make 4 different set ups of the proportion.

U=upper
B=bottom
L=left
R=right
W=whole

$\frac{UL}{BL} = \frac{UR}{BR}$ $\left(\frac{12}{16} = \frac{15}{x}\right)$ $\frac{UL}{UR} = \frac{BL}{BR}$ $\left(\frac{12}{15} = \frac{16}{x}\right)$

$\frac{UL}{WL} = \frac{UR}{WR}$ $\left(\frac{12}{28} = \frac{15}{15+x}\right)$ $\frac{BL}{WL} = \frac{BR}{WR}$ $\left(\frac{16}{28} = \frac{x}{15+x}\right)$

☆ depending on situation, use different proportions.

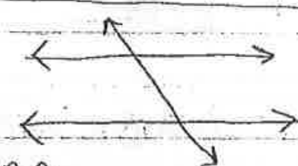
- For this example: (why?) single x

$\left(\frac{UL}{BL} = \frac{UR}{BR}\right)$ $\frac{12}{16} = \frac{15}{x}$

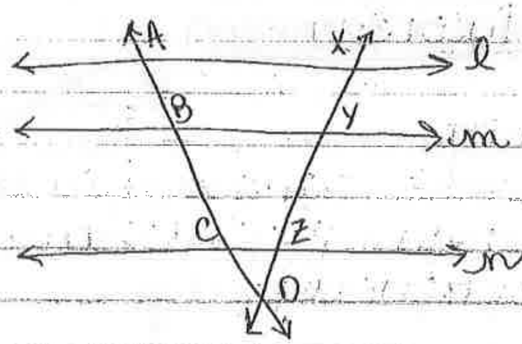
$16 \cdot 15 = 12 \cdot x$
$240 = 12x$
$x = 20$

- Two-Transversal Proportionality Corollary

- Recall transversal



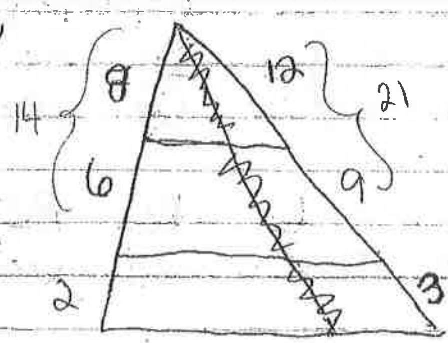
- If there are two intersecting transversals, three or more parallel lines cut the transversal proportionally.



You can write the proportions just like in the side-splitting theorem

$\frac{AB}{BC} = \frac{XY}{YZ}$ $\frac{AB}{AC} = \frac{XY}{XZ}$ $\frac{AB}{XY} = \frac{BC}{YZ}$ $\frac{BC}{AC} = \frac{YZ}{XZ}$

Ex: 2/



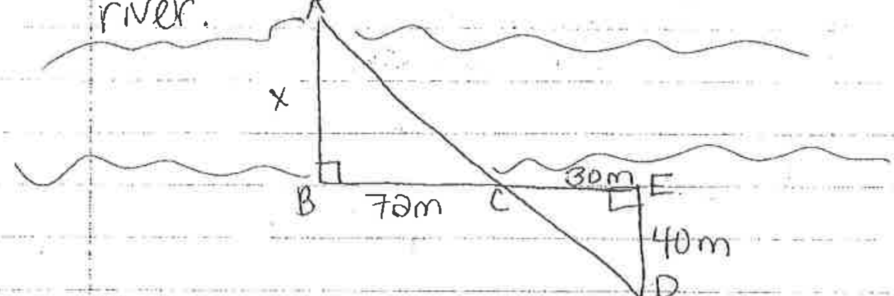
$\frac{8}{6} = \frac{12}{x}$ $\frac{4}{3}$ $72 = 8x$
$9 = x$

$\frac{14}{x} = \frac{21}{3}$ $14 \cdot 3 = 21x$
$42 = 21x$
$2 = x$

⑬

8.5 Indirect Measurement and Additional Similarity Theorems

Warm-up: Determine the distance across the river.



The triangles are similar because $\angle B = \angle E$ and $\angle ACB \cong \angle ECD$ (vertical angles)

So,

$$\frac{AB}{DE} = \frac{BC}{CE} \qquad \frac{x}{40} = \frac{72}{30} \qquad \frac{x}{40} = \frac{12}{5}$$

$40 \cdot 72 = 30x$ $\qquad$ $5x = 12 \cdot 40$

$2880 = 30x$ $\qquad$ $5x = 480$

$96 = x$ $\qquad$ $x = 96$

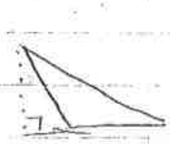
Proportional Altitudes Theorem

- recall altitude (there are 3 altitudes per triangle)

from one vertex perpendicular to other side

- altitude can be:
  - inside (above)

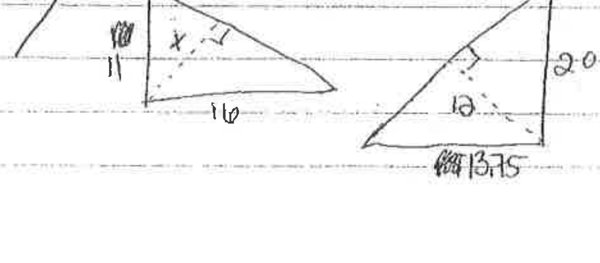
outside:



on the side:

- The ratio of corresponding altitudes is the same as the ratio of proportional sides.

Ex: 1



altitudes

$\frac{x}{12} = \frac{11}{13.75}$ or $\frac{x}{12} = \frac{16}{20}$

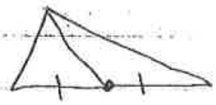
$13.75x = 11 \cdot 12$ $\qquad$ $20x = 12 \cdot 16$

$13.75x = 132$ $\qquad$ $20x = 192$

$x = 9.6$ $\qquad$ $x = 9.6$

Proportional Medians Theorem

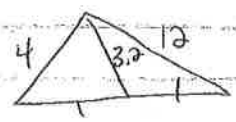
- recall median (there are three medians per triangle)



from vertex to other side, cuts in half

- If two triangles are similar, their corresponding medians have the same ratio as corresponding sides

Ex: 2/



$\frac{4}{6} = \frac{3.2}{x}$ $\quad$ $\frac{2}{3} = \frac{3.2}{x}$

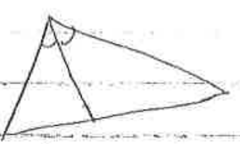
$4x = 6 \cdot 3.2$ $\quad$ $2x = 3 \cdot 3.2$

$4x = 19.2$ $\quad$ $2x = 9.6$

$x = 4.8$ $\quad$ $x = 4.8$

Proportional Angle Bisectors Theorem

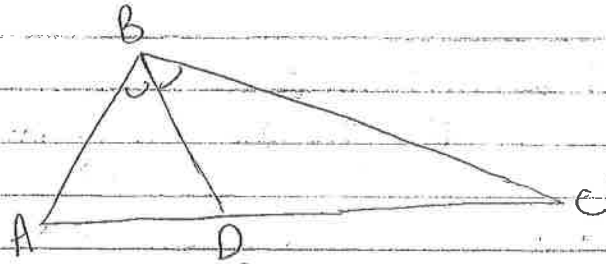
- recall angle bisectors
- Similar triangles have corresponding angle bisectors with the same ratio as corresponding sides



Proportional Segments Theorem

- ratio of the split parts (from an angle bisector) is equal to the ratio of the others sides of the triangle.

Ex: 3/



$\frac{AD}{DC} = \frac{AB}{BC}$

HW: 8.5A all