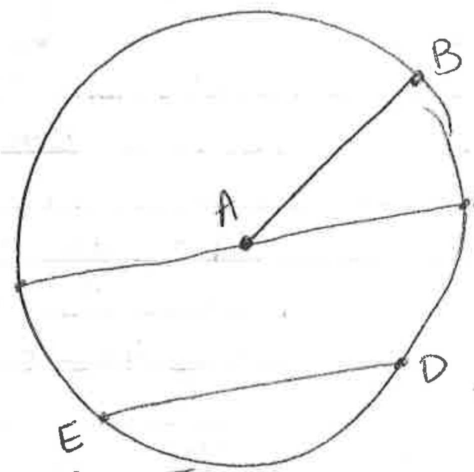
9.1 Chords and Arcs

• recall circle: set of points equidistant from a center

ex:



★ write radii, diameter & chords as segments

radii $\overline{AB}$, $\overline{AF}$, $\overline{AC}$ diameter $\overline{CF}$

Chords: a segment whose endpoints are on a circle

$\overline{CF}$, $\overline{DE}$ (diameter is a chord through center)

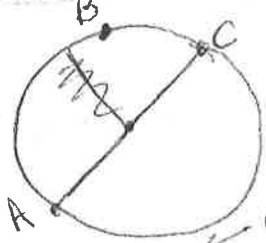
- Major & Minor Arcs

ASSET arc - an unbroken part of a circle (on the part of the circumference) circumference $= 2\pi r$ or πd

semicircle - 180° arc (half the circle's circumference)

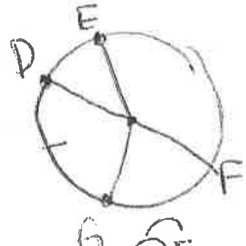
minor arc - less than 180° arc

major arc - greater than 180° arc



$\overset{\frown}{ABC}$ semicircle

arc symbol named w/ endpoints and one other letter



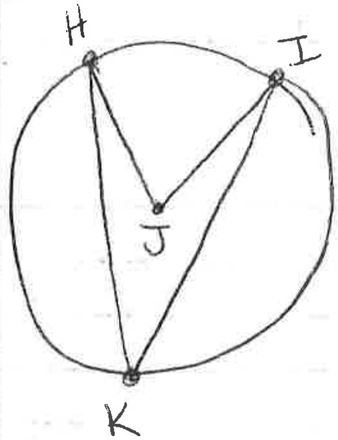
$\overset{\frown}{DE}$ minor arc only endpoints

$\overset{\frown}{DFG}$ major arc

degree measure of all arcs added = 360°

arc measure - arcs measured by their corresponding central angle

Central & Inscribed Angles



central angle - endpoints on circle vertex at center : $\angle HJI$

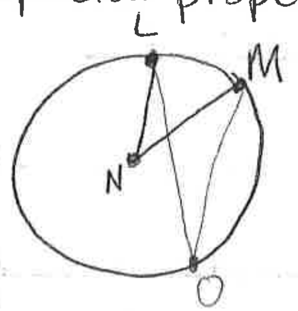
inscribed angle - endpoints and vertex on ~~circ~~ circle : $\angle HKI$

intercepted arc - arc made by the endpoints of a central or inscribed angle : $\widehat{HI}$

Save notes for 9.3 w/ examples

★ Special properties of inscribed / central angles:

Check to see if the vertex of inscribed can be on sides of circle
It can



- ~~inscribed~~ intercepted arc and central angle = same degree measure
- ~~inscribed~~ intercepted arc is double the inscribed angle:
  - $\widehat{LM} = 2 \cdot \angle LOM$

Arc Length

★ NOT THE SAME AS DEGREE MEASURE OF AN ARC!! ★

$m\widehat{AC}$ typically means degree measure
you will see length of $\widehat{AC}$ for ~~arc~~ arc length

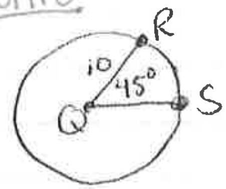
Arc length = L ; radius = r ; M = degree measure of arc

what part of circle is that arc of the circumference

$$L = \frac{M}{360^\circ} \cdot (2\pi r)$$ (usually measured as $\#\pi$)

9.1 Cont'd

Ex: 1/



a. Find $m\overset{\frown}{RS}$. 45°

b. Find length of $\overset{\frown}{RS}$.

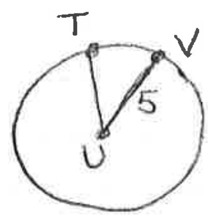
$$L = \frac{M}{360}(2\pi r)$$

$$L = \frac{45}{360}(2\pi(10))$$

$$= \frac{900}{360}\pi$$

$$= \frac{5\pi}{2}$$

Ex: 2/



Given: length of $\overset{\frown}{TV} = \pi$

Find $m\overset{\frown}{TV}$.

$$L = \frac{M}{360}(2\pi r)$$

$$360 \cdot \pi = \frac{M}{\cancel{360}}(2\pi(5)) \cdot \cancel{360}$$

$$\frac{360\pi}{10\pi} = \frac{10\pi M}{10\pi}$$

$$36^\circ = M$$

$\boxed{\text{HW: Pg. 570 \# 11-38}}$

ok for these

not 12 B~ 30

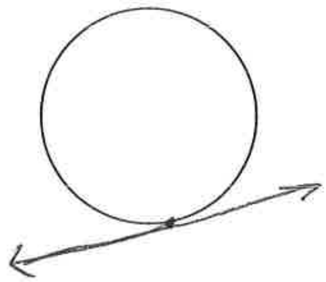
instead 9.3 HW (Rodgers) adapted

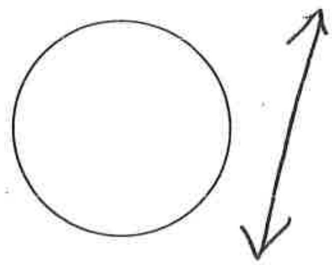
9.2 SECANT AND TANGENT LINES

Lines and Circles

There are three ways that lines can relate to a circle.

Two Points of Intersection	One Point of Intersection	Zero Points of Intersection
secant line	*tangent line*	



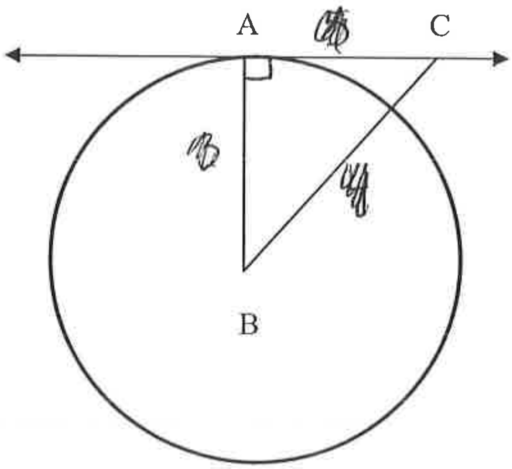


* Kuta Arcs & Chords all

Properties of Secant and Tangent Lines

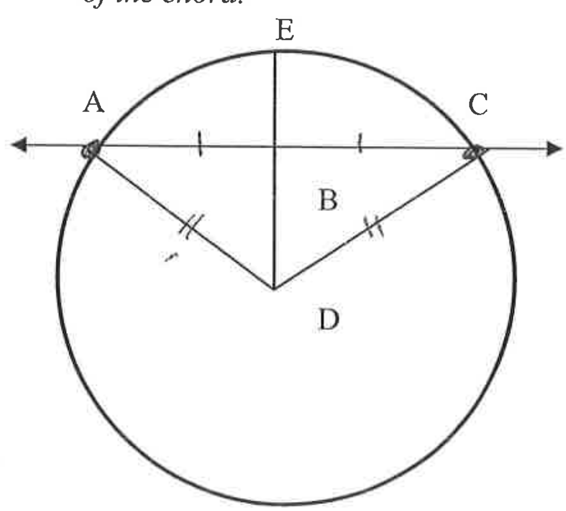
Radii and Tangent Lines

If a line is tangent to a circle, then the line is perpendicular to the radius through the point of tangency.

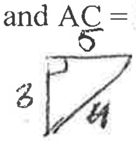


Radii and Secant Lines

A radius that is perpendicular to a chord of a circle is the bisector of the chord.



1. If AB = 3 and AC = 5, BC = $\sqrt{34}$

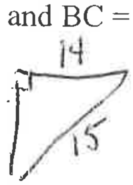


$9+25=c^2$

$34=c^2$

$\sqrt{34}=c$

2. If AC = 14 and BC = 15, what is the diameter?



$a^2+14^2=15^2$

$a^2+196=225$

$a^2=29$

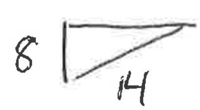
$a=\sqrt{29}$

$d=2\sqrt{29}$

1. Name a segment congruent to AB BC

2. Name segments congruent to AD DC

3. If BD = 8 and DC = 14, AB = $\sqrt{132}$



$8^2+b^2=14^2$

$64+b^2=196$

$b^2=132$

$b=\sqrt{132}$

9.4.

ANGLES FORMED BY SECANTS AND TANGENTS

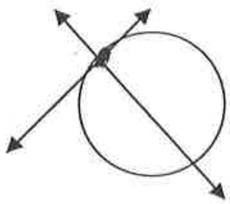
There are three places where secant and tangent lines can intersect about a circle:

on, in, or outside of a circle.

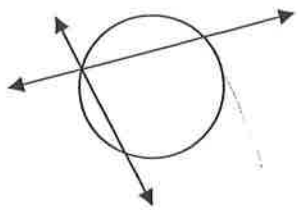
When the vertex is on the circle:

2·angle = arc

Case 1: Secant and Tangent



Case 2: Two Secants

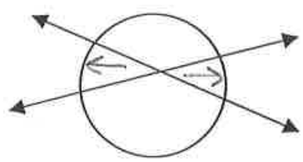


If a tangent and a secant or two secants intersect on a circle, then the measure of the angle formed is ½ *the measure of the intercepted arc.*

When the vertex is in the circle:

2·angle = arc + arc

Case 1: Two Secants

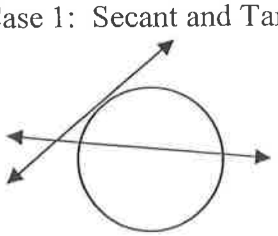


If two secants intersect inside of a circle, the measure of the angle formed is ½ the sum ~~of~~ *of the intercepted arcs.*

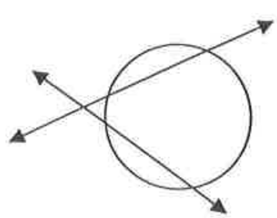
When the vertex is outside of the circle:

2·angle = lg arc − sm arc

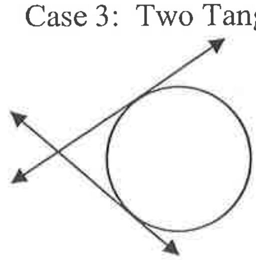
Case 1: Secant and Tangent



Case 2: Two Secants



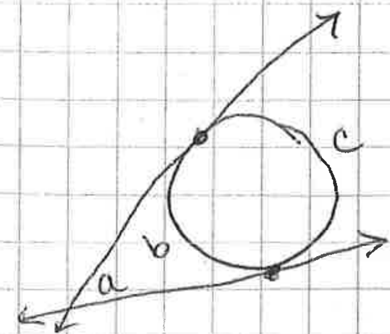
Case 3: Two Tangents



If a secant and a tangent, two secants, or two tangents intersect outside of a circle, the measure of the angle is ½ the difference *of the intercepted arcs.*

9.4 Fucsko's Theorem

- Vertex is outside of the circle
- Case 3: Two tangents **ONLY**



Original Formula:

$$a = \frac{1}{2}(c - b)$$

Well, $c = 360 - b$, so substitute & simplify:

$$a = \frac{1}{2}(360 - b - b)$$

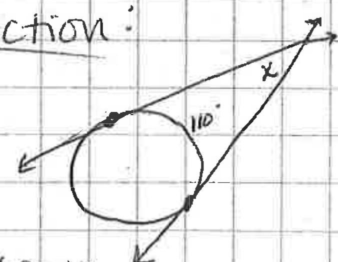
$$a = \frac{1}{2}(360 - 2b)$$

$$a = 180 - b$$

$$+b \qquad +b$$

Fucsko's Theorem: $a + b = 180°$!!!

In Action:



Old Way

$x = \frac{1}{2}(m - 110)$

$x = \frac{1}{2}(250 - 110)$

$x = \frac{1}{2}(140)$

$x = 70°$

$m = 360 - 110$

$m = 250$

Fucsko's Theorem

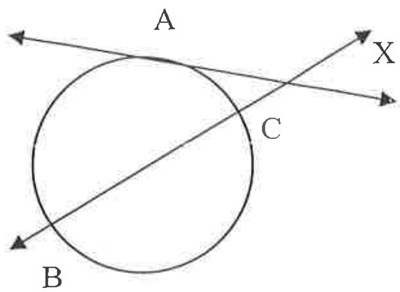
$110 + x = 180$

$x = 70°$

9.5

SEGMENTS OF TANGENTS, SECANTS, AND CHORDS

Segment Relationships in Circles



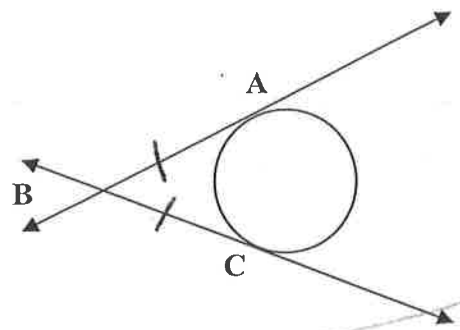
Tangent Segment: $\overline{XA}$

Secant Segment: XB

External Secant Segment: $\overline{XC}$

Chord: $\overline{BC}$

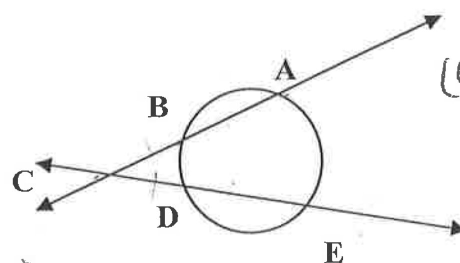
Segments Formed by Tangents



If two segments are tangent to a circle from the same external point, then the segments are equal.

Equation: $\overline{AB} = \overline{BC}$

Segments Formed by Secants

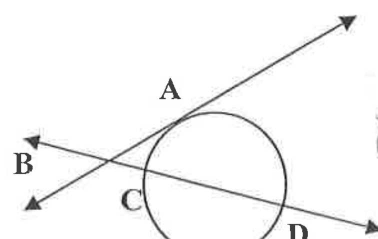


outside x whole = outside x whole

$(CB)(CA)=(CD)(CE)$

If two secants intersect outside of a circle, the product of the lengths of the secant segments and their external segments are equal.

Equation: Whole x Outside = Whole x Outside



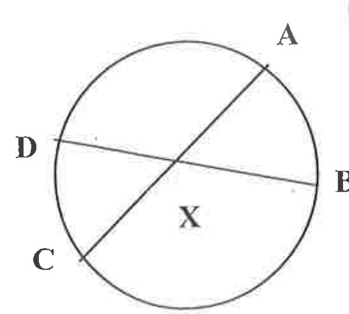
outside x whole = tangent^2

$(BC)(BD)=(BA)^2$

If a secant and a tangent intersect Outside of a circle, then the product of The secant segment and its external Segment equals the tangent segment squared.

Equation: Whole x Outside = Tan^2

Segments Formed by Intersecting Chords



Part A x part B = part A x part B

$(DX)(XB)=(AX)(XC)$

If two chords intersect inside of a circle, Then the product of the lengths of the Segments of the chord equal each other.

Equation: Part x Part = Part x Part

Mrs. Fucsko - Geometry

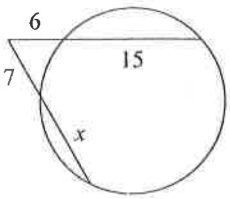
Name Examples-Key

9.5 Segment Lengths in Circles

Date____________

Solve for x. Assume that lines which appear tangent are tangent.

1)



$6(6+15)=7(7+x)$

$6(21)=49+7x$

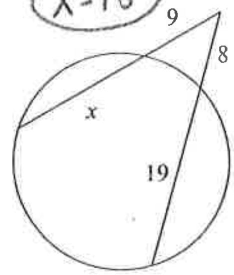
$126=49+7x$

$-49 \quad -49$

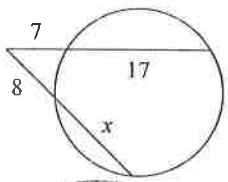
$77=7x$

$x=11$

2) $x=15$

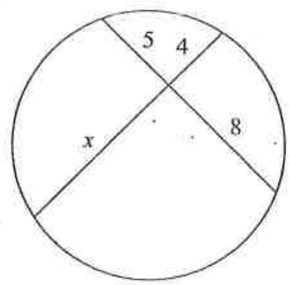


3)



$x=13$

4)

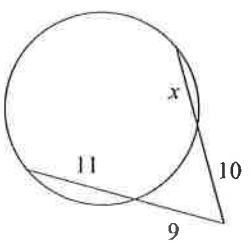


$5\cdot 8=4\cdot x$

$40=4x$

$x=10$

5)



$10(10+x)=9(9+11)$

$100+10x=9(20)$

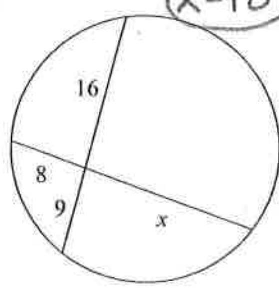
$100+10x=180$

$-100 \quad -100$

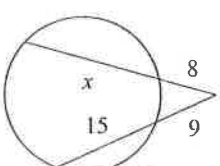
$10x=80$

$x=8$

6) $x=18$

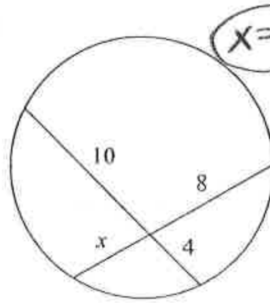


7)

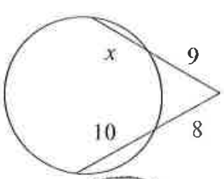


$x=19$

8) $x=5$

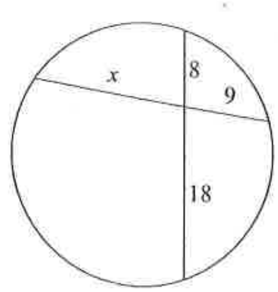


9)



$x=7$

10)

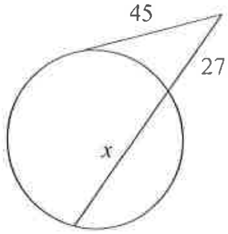


$8(18)=9x$

$144=9x$

$x=16$

11)



$27(27+x)=45^2$

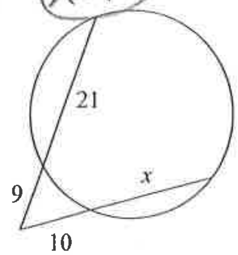
$729+27x=2025$

$-729 \qquad -729$

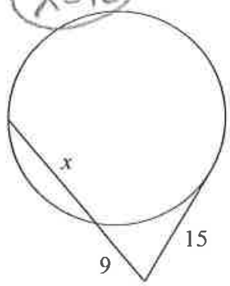
$27x=1296$

$x=48$

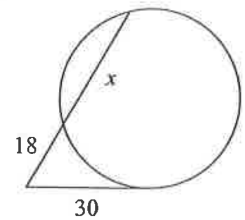
12) $x=17$



13) $x=16$



14)



$18(18+x)=30^2$

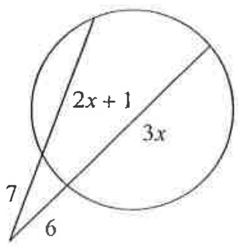
$324+18x=900$

$-324 \qquad -324$

$18x=576$

$x=32$

23 ~~15~~)



$7(7+2x+1)=6(6+3x)$

$7(8+2x)=36+18x$

$56+14x=36+18x$

$-14x \qquad -14x$

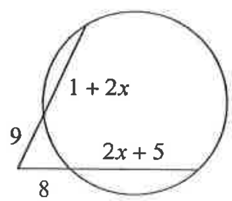
$56=36+4x$

$-36 \quad -36$

$20=4x$

$x=5$

24 ~~16~~)



$9(9+1+2x)=8(8+2x+5)$

$9(10+2x)=8(13+2x)$

$90+18x=104+16x$

$-16x \qquad -16x$

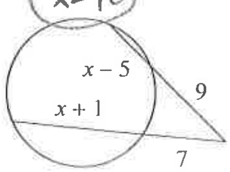
$90+2x=104$

$-90 \qquad -90$

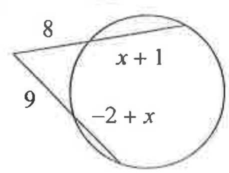
$2x=14$

$x=7$

25 ~~17~~) $x=10$



26 ~~18~~) $x=9$



9.6 Graphing Circles

- Equation for a circle (center at zero)

$$x^2 + y^2 = r^2$$

- Center at (h,k)

$$(x-h)^2 + (y-k)^2 = r^2$$

So, the center is "opposite" of what it looks like...

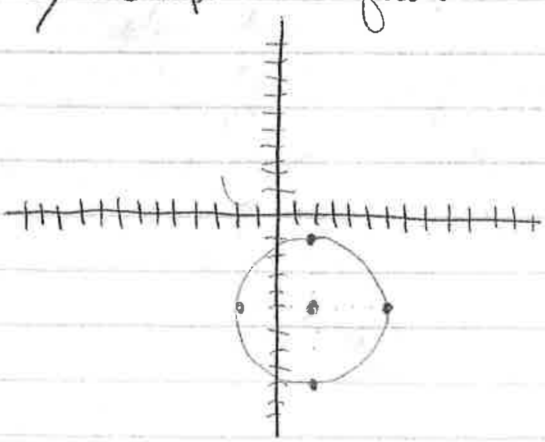
Ex: 1/ $(x+2)^2 + (y-3)^2 = 25$
center $(-2,3)$ radius 5

Ex: 2/ Write an equation for a circle with a center at $(6,-8)$ and a radius of 7.

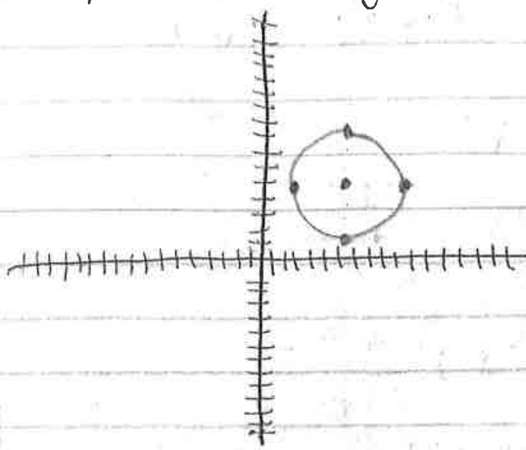
$$(x-6)^2 + (y+8)^2 = 49$$

- Graphing

Ex: 3/ Graph the equation $(x-2)^2 + (y+5)^2 = 16$
center $(2,-5)$
radius 4



Ex: 4/ write the equation of the circle.



center (7, 5)

radius 3

$$(x-7)^2+(y-5)^2=9$$