Derivative Review

Name: Key

Find the first derivative of each of the following:

1. $f(x) = \ln(x^3 + 2x + 18)$ chain rule

$f'(x) = \frac{1}{x^3+2x+18} \cdot 3x^2+2$

$$\boxed{f'(x) = \frac{3x^2+2}{x^3+2x+18}}$$

2. $f(x) = e^{(3x^2+2x+9)}$ chain rule

$f'(x) = e^{(3x^2+2x+9)} \cdot (6x+2)$

$$\boxed{f'(x) = (6x+2)e^{(3x^2+2x+9)}}$$

quotient rule

3. $f(x) = \frac{\cos x}{x^2+1}$

$f'(x) = -\sin(x)$

$g'(x) = 2x$

$f'(x) = \frac{-\sin(x)(x^2+1) - \cos(x)(2x)}{(x^2+1)^2}$

$f'(x) = \frac{-\sin(x)(x^2+1) - 2x\cos(x)}{(x^2+1)^2}$

$$\boxed{f'(x) = \frac{-x^2\sin x - \sin x - 2x\cos x}{(x^2+1)^2}}$$

Product rule

4. $y = x \ln x$

$f'(x) = 1 \quad g'(x) = \frac{1}{x}$

$y' = 1 \cdot \ln x + x \cdot \frac{1}{x}$

$$\boxed{y' = \ln x + 1}$$

$f'(\theta) = e^{\theta}$

$g'(\theta) = -\csc^2\theta$

5. $h(\theta) = \csc\theta + \underbrace{e^{\theta}\cot\theta}_{\text{productrule}}$

$-\csc\theta\cot\theta + e^{\theta}\cot\theta + e^{\theta}(-\csc^2\theta)$

$\boxed{h'(\theta) = -\csc\theta\cot\theta + e^{\theta}\cot\theta - e^{\theta}\csc^2\theta}$

6. $y = x^3 - 4x + 6$

$\boxed{y' = 3x^2 - 4}$

7. $y = \ln(\cosh x)$ chain rule

$y' = \frac{1}{\cosh x} \cdot (+)\sinh x$

$y' = +\frac{\sinh x}{\cosh x}$

$\boxed{y' = +\tanh x}$

8. $y = \arctan\sqrt{x}$ chain rule

$y = \arctan x^{1/2}$

$y' = \frac{1}{1+(\sqrt{x})^2} \cdot \frac{1}{2}x^{-1/2}$

$y' = \frac{1}{(1+x)} \cdot \frac{1}{2\sqrt{x}}$

$\boxed{y = \frac{1}{2\sqrt{x}(1+x)} = \frac{1}{2\sqrt{x} + 2x\sqrt{x}}}$

9. Product & chain

$\sin(xy) = x^2 - y^2$

product

$\cos(xy)(y + xy') = 2x - 2yy'$

$y\cos(xy) + xy'\cos(xy) = 2x - 2yy'$

$xy'\cos(xy) + 2yy' = 2x - y\cos(xy)$

$y'(x\cos(xy) + 2y) = 2x - y\cos(xy)$

$\boxed{y' = \frac{2x - y\cos(xy)}{x\cos(xy) + 2y}}$

10. $xy^4 + x^2y = x + 3y$

product product

$\underbrace{y^4 + x\cdot(4y^3y')} + \underbrace{2xy + x^2y'} = 1 + 3y'$

$y^4 + 4xy^3y' + 2xy + x^2y' = 1 + 3y'$

$4xy^3y' + x^2y' - 3y' = 1 - y^4 - 2xy$

$y'(4xy^3 + x^2 - 3) = 1 - y^4 - 2xy$

$\boxed{y' = \frac{1 - y^4 - 2xy}{4xy^3 + x^2 - 3}}$

11. Use logarithmic differentiation

$y = x^{\tan x}$

$\ln y = \ln(x^{\tan x})$

$\ln y = \underbrace{\tan x \ln x}$ product

$\frac{1}{y} \cdot y' = \sec^2(x) \ln x + \tan x \cdot \frac{1}{x}$

$\frac{y'}{y} = \sec^2(x) \ln x + \frac{\tan x}{x}$

$y' = (x^{\tan x})(\sec^2(x) \ln x + \frac{\tan x}{x})$

12. $xy = 2x^2y$ product rule

$y + xy' = 4xy + 2x^2y'$

$xy' - 2x^2y' = 4xy - y$

$y'(x - 2x^2) = 4xy - y$

$\boxed{y' = \frac{4xy - y}{x - 2x^2}}$

13. $f(x) = 4x^5 - \frac{2}{x^3} - 2\pi = 4x^5 - 2x^{-3} - 2\pi$

$f'(x) = 20x^4 + 6x^{-4}$

$\boxed{f'(x) = 20x^4 + \frac{6}{x^4}}$

14. $f(x) = \underbrace{e^{3x} \cos(2x)}_{\text{product}}$ chain

$f'(x) = e^{3x} \cdot 3$

$g'(x) = -\sin(2x) \cdot 2$

$f'(x) = 3e^{3x} \cos(2x) + e^{3x}(-\sin(2x) \cdot 2)$

$f'(x) = 3e^{3x} \cos(2x) - 2e^{3x} \sin(2x)$

or

$f'(x) = e^{3x}(3\cos(2x) - 2\sin(2x))$

15. $f(x) = \tan^2(3x) = \underbrace{[\underbrace{\tan(3x)}_{\text{chain}}]^2}_{\text{Chain}}$

$f'(x) = 2\tan(3x) \cdot \sec^2(3x) \cdot 3$

$\boxed{f'(x) = 6\tan(3x)\sec^2(3x)}$

16. $f(x) = e^{\ln(3x-7)} = 3x - 7$

$\boxed{f'(x) = 3}$

Find the requested derivative:

17. The second derivative of:

$$f(x) = \underbrace{x^4 e^x}_{\text{product}}$$

$$f'(x) = \underbrace{4x^3 e^x}_{\text{product}} + \underbrace{x^4 e^x}_{\text{product}}$$

$$f''(x) = \underbrace{12x^2e^x + 4x^3e^x}\; + \underbrace{4x^3e^x + x^4e^x}$$

$$f''(x) = e^x(12x^2 + 4x^3 + 4x^3 + x^4)$$

$$\boxed{f''(x) = e^x(x^4 + 8x^3 + 12x^2)}$$

or

$$\boxed{f''(x) = x^2e^x(x^2 + 8x + 12)}$$

18. The 4th derivative of:

$$f(x) = x^3 + 6x^2 + 6x + 7$$

$$f'(x) = 3x^2 + 12x + 6$$

$$f''(x) = 6x + 12$$

$$f'''(x) = 6$$

$$\boxed{f^{(4)}(x) = 0}$$

19. The third derivative of:

$$y = \sin x$$

$$y' = \cos(x)$$

$$y'' = -\sin(x)$$

$$\boxed{y''' = -\cos(x)}$$

20. All higher order derivatives of:

$$f(x) = x^4 - \tfrac{2}{3}x^3 + 4x - 2$$

$$f'(x) = 4x^3 - 2x^2 + 4$$

$$f''(x) = 12x^2 - 4x$$

$$f'''(x) = 24x - 4$$

$$f^{(4)}(x) = 24$$

$$f^{(5)}(x) = 0$$

Multiple Choice:

21. The derivative of $f(x) = x^2 \arctan(x)$ will be

(a) $\frac{2x}{1+x^2}$

(b) $\frac{x^2}{1+x^2} + 2x\arctan(x)$

(c) $2x \arctan x$

(d) $\frac{x^2}{\sqrt{1-x^2}} + 2x\arctan(x)$

(e) None of these

$2x(\arctan(x)) + x^2 \cdot \frac{1}{1+x^2}$

22. The derivative of $h(y) = (5y^4 + e^y)^9$ will be

(a) $9(20y^3 + e^y)^8$

(b) $9(5y^4 + e^y)^8$

(c) $9(20y^3 + e^y)^8(20y^3 + ye^{y-1})$

(d) $9(5y^4 + e^y)^8(20y^3 + e^y)$

(e) None of these

$h'(y) = 9(5y^4 + e^y)^8 \cdot (20y^3 + e^y)$

23. The derivative of $y = 6^x - \frac{1}{\sqrt[3]{x}} - 6x$ will be

$x^{-1/3}$

(a) $x6^{x-1} + \frac{1}{3x^{4/3}} - 6$

(b) $6^x(\ln 6) + \frac{1}{3x^{4/3}} - 6$

(c) $6^x(\ln 6) + \frac{1}{3\sqrt{x}} - 6$

(d) $x6^{x-1}(\ln 6) + \frac{1}{3\sqrt{x}} - 6$

(e) None of these

$y' = 6^x \ln(6) + \frac{1}{3}x^{-4/3} - 6$

$= 6^x \ln(6) + \frac{1}{3\sqrt[3]{x^4}} - 6$

$= 6x \ln(6) + \frac{1}{3x\sqrt[3]{x}} - 6$

24. The derivative of $g(\alpha) = 12e^{-4\alpha} - e^{-\alpha}$ will be

chain chain

(a) $48e^{-4\alpha} - e^{-\alpha}$

(b) $-48e^{-4\alpha} + e^{-\alpha}$

(c) $48e^{-4\alpha-1} - e^{-\alpha-1}$

(d) $-48e^{-4\alpha-1} + e^{-\alpha-1}$

(e) None of these

$g(\alpha) = 12e^{-4\alpha} \cdot (-4) - e^{-\alpha} \cdot (-1)$

$g(\alpha) = -48e^{-4\alpha} + e^{-\alpha}$

Applications:

25. Find the equation of the tangent line at $(-1, 2)$ to the curve given by

$$x^2 + \underbrace{xy^2} + y^3 = 5$$

$$2x + y^2 + x \cdot (2yy') + 3y^2y' = 0$$

$$2xyy' + 3y^2y' = -y^2 - 2x$$

$$y'(2xy + 3y^2) = -y^2 - 2x$$

$$y' = \frac{-y^2 - 2x}{2xy + 3y^2} = \frac{-(2)^2 - 2(-1)}{2(-1)(2) + 3(2)^2} = \frac{-2}{8} = \frac{-1}{4}$$

$y = mx + b$

$2 = -\frac{1}{4}(-1) + b$

$2 = \frac{1}{4} + b$

$\frac{7}{4} = b$

$$\boxed{y = \frac{-1}{4}x + \frac{7}{4}}$$

26. Consider $f(x) = x^3 - 6x^2 + 9x + 3$. Find all critical point(s) of $f(x)$. On what interval(s) is(are) $f(x)$ increasing? On what interval(s) is(are) $f(x)$ decreasing?

$f'(x) = 3x^2 - 12x + 9 = 0$

$3(x^2 - 4x + 3) = 0$

$3(x-3)(x-1) = 0$

$x = 3 \quad x = 1$

1, 3

$f'(0)$ $+$ $f'(2)$ $-$ $f'(4)$ $+$

Increasing $(-\infty, 1) \cup (3, \infty)$
Decreasing $(1, 3)$

27. Find the equation of the tangent line to the curve $y = \sqrt[4]{x}$ at the point (1,1).

$y = x^{1/4}$

$y' = \frac{1}{4}x^{-3/4}$

$y'\big|_{x=1, y=1} = \frac{1}{4}(1)^{-3/4} = \frac{1}{4}$

$y = 1 + \frac{1}{4}(x-1)$

$y = mx + b$

$1 = \frac{1}{4}(1) + b$

$\frac{3}{4} = b$

$$\boxed{y = \frac{1}{4}x + \frac{3}{4}}$$

28. Suppose that $F(x) = f(x)g(x)$ and $h(x) = f(g(x))$, where

- $f(2) = 4$
- $g(2) = 3$
- $g'(2) = 3$
- $f'(2) = 1$
- $f'(3) = 5$

Find

(a) $F'(2)$

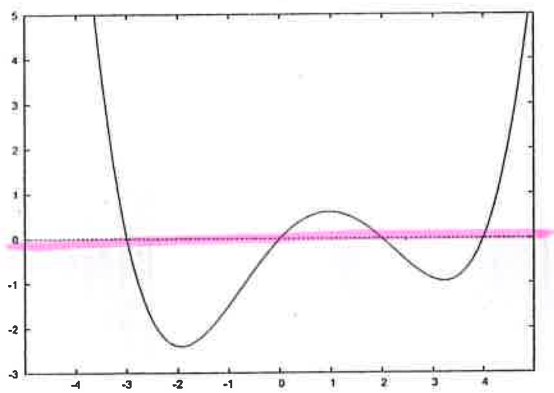
(b) $h'(2)$

a) $F'(2) = f'(x)g(x) + f(x)g'(x)$
$= 1 \cdot 3 + 4 \cdot 3 = 15$

b) $h'(2) = f'(g(x)) \cdot g'(x)$
$= f'(3) \cdot g'(x)$
$= 5 \cdot 3 = 15$

29. The graph of the **derivative** of a function f is shown below (This is not the graph of f)

Based on this graph, find the intervals where f is increasing



increasing
$(-\infty, -3) \cup (0, 2) \cup (4, \infty)$

30. If a ball is thrown vertically upward with a velocity of 80 ft/s, then its height after t seconds is $s = 80t - 16t^2$. What is the max height reached by the ball? What is the velocity of the ball when it is 96 ft above the ground and on its way down? At what time does the ball hit the ground and what is it's velocity?

a. height $s = 80t - 16t^2$
max height $y =$ (vertex)

$x = \frac{-b}{2a} = \frac{-80}{2(-16)} = 2.5$

$y = 80(2.5) - 16(2.5)^2$
$y = 200 - 100 = \boxed{100 \text{ ft}}$

b. velocity $s' = 80 - 32t$
orig. $96 = 80t - 16t^2$
$16t^2 - 80t - 96 = 0$
$t^2 - 5t + 6 = 0$
$(t-3)(t-2) = 0$
@96ft $t = 2, t = 3$
on it's way down

$s'(3) = 80 - 32(3)$
$= 80 - 96$
$= \boxed{-16 \text{ ft/s}^2}$

c. $0 = 80t - 16t^2$
$0 = 16t(5-t)$
$t = 0$ $\boxed{t = 5}$

$s'(5) = 80 - 32(5)$
$= 80 - 160$
$= \boxed{-80 \text{ ft/s}^2}$