College Algebra CHS

Name Key

Review - Unit 2

Date

Determine if the given relation is a function.

1) $(3, 3)\ (-1, 3)\ (1, -3)$ yes

Determine if the given equation is a function.

2) $y = \frac{x}{4} + \frac{x^2}{5}$ yes

3) $y^2 = 4x + 1$ no

4) $y = |x|$ yes

Evaluate each function.

5) $h(n) = |n| - 3$; Find $h(2)$ $= -1$

6) $w(a) = \frac{1}{2}a + \frac{4}{3}$; Find $w\left(-\frac{5}{7}\right)$

$\frac{1}{2}\left(-\frac{5}{7}\right) + \frac{4}{3} = \frac{41}{42}$

7) $k(n) = -2n^3 - 1$; Find $k(-x)$

$-2(-x)^3 - 1$

$-2(-x^3) - 1$

$2x^3 - 1$

8) $f(a) = a^3 + 3a^2$; Find $f(a - 3)$

$(a-3)(a-3)(a-3) + 3(a-3)^2$

$(a-3)(a^2-6a+9) + 3(a^2-6a+9)$

$a^3 - 6a^2 + 9a - 3a^2 + 18a \overset{-27}{\cancel{\ }} + 3a^2 - 18a + 27$

$a^3 - 6a^2 + 9a$

9) $p(x) = 2x + \frac{2}{3}$; Find $p\left(x + \frac{1}{2}\right)$

$2\left(x + \frac{1}{2}\right) + \frac{2}{3}$

$2x + 1 + 2/3$

$2x + 5/3$

10) $f(n) = n^2 - n$; Find $f\left(\frac{n}{3}\right)$

$\left(\frac{n}{3}\right)^2 - \left(\frac{n}{3}\right)$

$\frac{n^2}{9} - \frac{n}{3}$ or $\frac{n^2 - 3n}{9}$

Compute the difference quotient for the given function.

11) $f(x) = 4x^2 - 6x$

$$\frac{4(x+h)^2 - 6(x+h) - (4x^2 - 6x)}{h}$$

$$\frac{4(x^2 + 2hx + h^2) - 6x - 6h - 4x^2 + 6x}{h}$$

4

$$\frac{\cancel{4x^2} + 8hx + 4h^2 \cancel{- 6x} - 6h \cancel{- 4x^2} \cancel{+ 6x}}{h}$$

$$\frac{h(8x + 4h - 6)}{h} = 8x + 4h - 6$$

12) $g(t) = t^2 + 3t - 4$

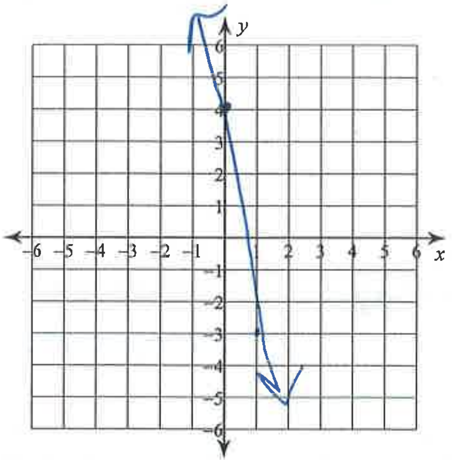
$(x+h)^2 + 3(x+h) - 4 - (t^2 + 3t - 4)$

$\cancel{t^2} + 2h\cancel{t} + h^2 \cancel{+ 3t} + 3h \cancel{- 4} \cancel{- t^2} \cancel{- 3t} \cancel{+ 4}$

$$\frac{h(2t + h + 3)}{h} = 2t + h + 3$$

Sketch the graph of each line.

13) $-7x + 4 = y$



Write the equation of a line in point-slope form and slope-intercept form that passes through the following points.

14) $(3, -1)\ (1, 5)$

$y+1 = -3(x-3)$ $\quad y+1 = -3x+9$

or $\quad y = -3x+8$

$\frac{5+1}{1-3} = \frac{6}{-2} = -3$ $\quad y-5 = -3(x-1)$ or $y-5 = -3x+3$

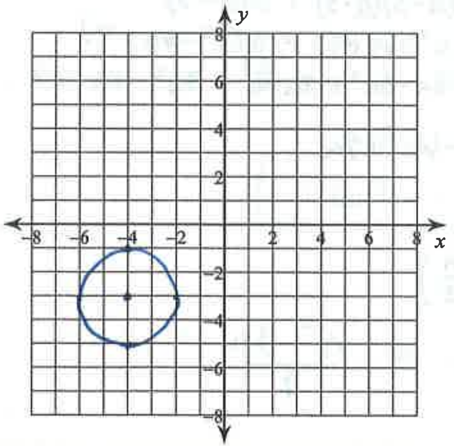
$y = -3x+8$

Identify the center and radius of each. Then sketch the graph.

15) $(x+4)^2 + (y+3)^2 = 4$

Center $(-4, -3)$

radius 2



Use the information provided to write the standard form equation of each circle.

16) $x^2 + y^2 + 8x - 28y + 203 = 0$

$x^2+8x\underline{+16} \quad +y^2-28y\underline{+196} = -203\underline{+16}\ \underline{+196}$

$(x+4)^2 + (y-14)^2 = 9$

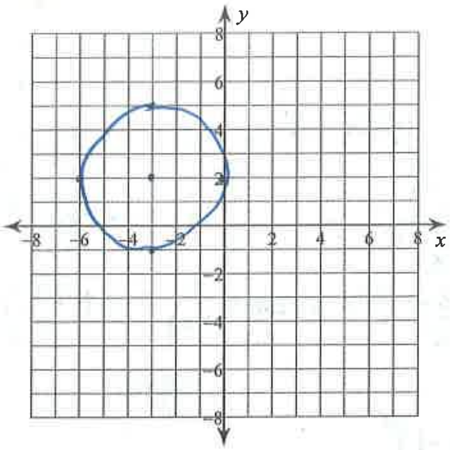
17) $x^2 + y^2 + 30x + 24y + 365 = 0$

$x^2+30x\underline{+225} \quad y^2+24y\underline{+144} = -365\ \underline{+225}\ \underline{+144}$

$(x+15)^2 + (y+12)^2 = 4$

Identify the center and radius of each. Then sketch the graph.

18) $x^2 + y^2 + 6x - 4y + 4 = 0$



$x^2+6x+9 \quad +y^2-4y+4 = -4+9+4$

$(x+3)^2+(y-2)^2 = 9$

radius = 3

Center = (-3, 2)

Perform the indicated operation.

19) $g(n) = 2n - 3$
$h(n) = 4n + 4$
Find $(g - h)(n)$

$2n-3-(4n+4)$
$2n-3-4n-4$
$-2n-7$

20) $g(a) = a^2 + 3a$
$f(a) = a - 4$
Find $(g - f)(-2a)$

a^2+2a+4
$(-2a)^2+2(-2a)+4$
$4a^2-4a+4$

21) $g(n) = 4n + 2$
$f(n) = n^3 - 4n^2$
Find $(g \cdot f)(n)$

$(n^3-4n^2)(4n+2)$
$4n^4-16n^3+2n^3-8n^2$
$4n^4-14n^3-8n^2$

22) $g(x) = x^3 - 5x^2$
$h(x) = 3x - 3$
Find $(4g + 4h)(x)$

$4(x^3-5x^2)+4(3x-3)$
$4x^3-20x^2+12x-12$

23) $f(n) = 4n - 1$
$g(n) = -n^2 + 3$
Find $(4f + 2g)(n)$

$4(4n-1)+2(-n^2+3)$
$16n-4-2n^2+6$
$-2n^2+16n+2$

24) $g(x) = 4x - 2$
$h(x) = -4x - 1$
Find $(5g + 5h)(x)$

$5(4x-2)+5(-4x-1)$
$20x-10-20x-5$
-15

25) $g(x) = x^2 + 2x$
Find $(g \circ g)(x)$

$(x^2+2x)^2+2(x^2+2x)$
$(x^2+2x)(x^2+2x)$
$x^4+4x^3+4x^2+2x^2+4x$
$x^4+4x^3+6x^2+4x$

26) $g(n) = n^3 + 5n^2$
$f(n) = -n + 4$
Find $(g \circ f)(n)$

$(-n+4)^3+5(-n+4)^2$
$(-n+4)(n^2-8n+16)+5(n^2-8n+16)$
$-n^3+8n^2-16n+4n^2-32n+64+5n^2-40n+8$
$-n^3+17n^2-88n+144$

27) $h(x) = x - 5$
$g(x) = x^2 - 2x$
Find $(h \circ g)(2)$

$(x^2-2x)-5$
$2^2-2(2)-5=-5$

28) $g(x) = 2x + 1$
Find $(g \circ g)(x - 1)$

$2(2x+1)+1$
$4x+2+1$
$4x+3$
$4(x-1)+3$
$4x-4+3 = 4x-1$

State if the given functions are inverses.

29) $h(n) = -2n + 1$

$f(n) = -\frac{1}{2}n + \frac{1}{2}$

$(h \circ f)(n) = -2\left(-\frac{1}{2}n + \frac{1}{2}\right) + 1$

$= n - 1 + 1$

$= n$

$(f \circ h)(n) =$

$= -\frac{1}{2}(-2n+1) + \frac{1}{2}$

$= n - \frac{1}{2} + \frac{1}{2}$

$= n$

yes

30) $g(x) = \frac{6 + \sqrt[3]{4x}}{2}$

$f(x) = 2(x-3)^3$

$(g \circ f)(x) = \frac{6 + \sqrt[3]{4(2(x-3)^3)}}{2}$

$= \frac{6 + \sqrt[3]{8(x-3)^3}}{2}$

$= \frac{6 + 2(x-3)}{2} = \frac{6 + 2x - 6}{2} = x$

$(f \circ g)(x) =$

$= 2\left(\frac{6 + \sqrt[3]{4x}}{2} - 3\right)^3$

$= 2\left(\frac{6 + \sqrt[3]{4x}}{2} - \frac{6}{2}\right)^3$

$= 2\left(\frac{6 + \sqrt[3]{4x} - 6}{2}\right)^3$

$= 2\left(\frac{\sqrt[3]{4x}}{2}\right)^3$

$= 2\left(\frac{4x}{8}\right) = \frac{8x}{8} = x$

yes

Find the inverse of each function.

31) $f(n) = 4n + 16$

$x = 4y + 16$

$x - 16 = 4y$

$f^{-1}(n) = \frac{n-16}{4}$

32) $g(n) = 3 + (n-1)^5$

$x = 3 + (y-1)^5$

$x - 3 = (y-1)^5$

$\sqrt[5]{x-3} = y - 1$

$g^{-1}(n) = \sqrt[5]{n-3} + 1$

33) $f(n) = -\frac{1}{n+2} - 3$

$x = -\frac{1}{y+2} - 3$

$\frac{x+3}{1} = \frac{-1}{y+2}$

$(y+2)(x+3) = -1$

$y + 2 = \frac{-1}{x+3}$

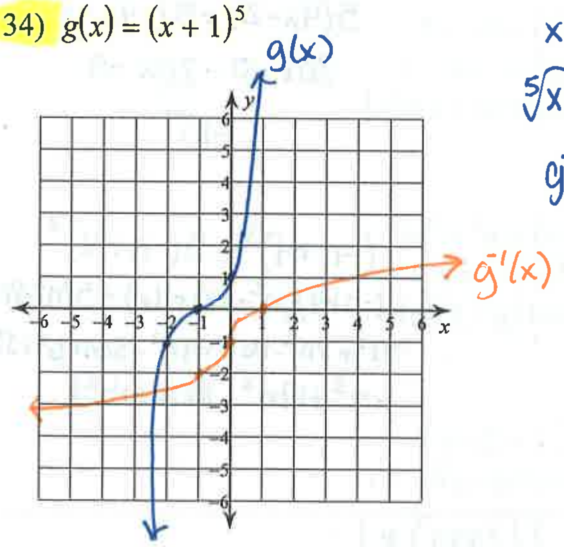
$f^{-1}(n) = \frac{-1}{n+3} - 2$

or

$f^{-1}(n) = \frac{-1-2(n+3)}{n+3} = \frac{-1-2n-6}{n+3} = \frac{-2n-7}{n+3}$

Find the inverse of each function. Then graph the function and its inverse.

34) $g(x) = (x+1)^5$



$x = (y+1)^5$

$\sqrt[5]{x} = y + 1$

$g^{-1}(x) = \sqrt[5]{x} - 1$

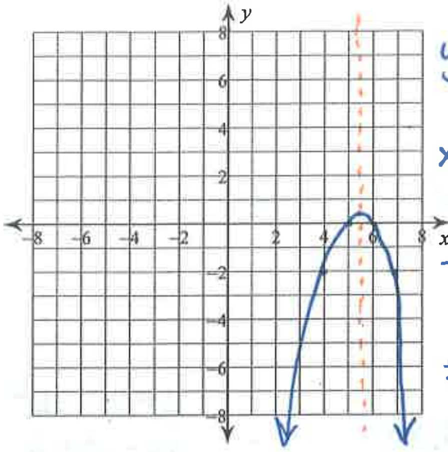
do not graph

Identify the vertex, axis of symmetry, y-intercept, and x-intercepts of each. Then sketch the graph.

35) $f(x) = -\left(x - \frac{11}{2}\right)^2 + \frac{1}{4}$

vertex $\left(\frac{11}{2}, \frac{1}{4}\right)$

aos $x = \frac{11}{2}$



y-intercept: $(0, -30)$

$-\left(0 - \frac{11}{2}\right)^2 + \frac{1}{4}$

x-intercepts: $(6,0)(5,0)$

$0 = -\left(x - \frac{11}{2}\right)^2 + \frac{1}{4}$

$-\frac{1}{4} = -\left(x - \frac{11}{2}\right)^2$

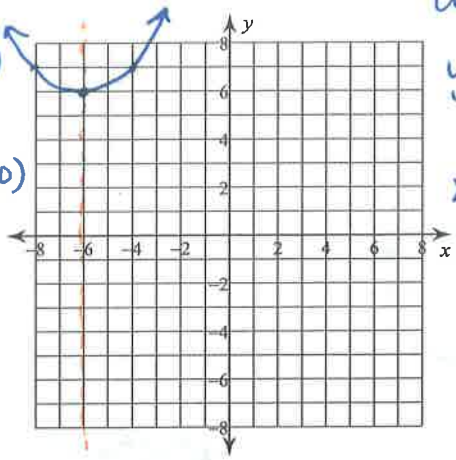
$\frac{1}{4} = \left(x - \frac{11}{2}\right)^2$

$\pm\frac{1}{2} = x - \frac{11}{2}$

36) $f(x) = \frac{1}{3}(x + 6)^2 + 6$

vertex $(-6, 6)$

aos $x = -6$



y-intercept: $(0, 18)$

$\frac{1}{3}(0+6)^2 + 6$

x-intercepts: none

Use the information provided to write the vertex form equation of each parabola. Then identify the vertex, axis of symmetry, y-intercept, and x-intercepts of each.

37) $y = 2x^2 + 8x + 18$

$y = 2(x^2 + 4x + \underline{4} - \underline{4}) + 18$

$y = 2[(x+2)^2 - 4] + 18$

$y = 2(x+2)^2 - 8 + 18$

$y = 2(x+2)^2 + 10$

vertex: $(-2, 10)$

aos: $x = -2$

y-intercept: $(0, 18)$

x-intercepts: none

38) $y = 18(x + 9)(x + 5)$

$y = 18(x^2 + 14x + 45)$

$y = 18(x^2 + 14x + \underline{49} - 49 + 45)$

$y = 18[(x+7)^2 - 4]$

$y = 18(x+7)^2 - 72$

vertex: $(-7, -72)$

aos: $x = -7$

y-intercept: $(0, 810)$

x-intercepts: $(-5,0)(-9,0)$

$18(x+7)^2 - 72 = 0$

$18(x+7)^2 = 72$

$(x+7)^2 = 4$

$x+7 = \pm 2$

39) $-10x^2 - 100x + y - 240 = 0$

$-10(x^2 + 10x + \underline{25} - 25) - 240 = -y$

$10[(x+5)^2 - 25] + 240 = y$

$10(x+5)^2 - 250 + 240 = y$

$10(x+5)^2 - 10 = y$

vertex: $(-5, -10)$

aos: $x = -5$

y-intercept: $(0, 240)$

x-intercepts: $(-4,0)(-6,0)$

$10(x+5)^2 = 10$

$(x+5)^2 = 1$

$x+5 = \pm 1$

40) $-2(y + 10) = (x + 3)^2$

$-2y - 20 = (x+3)^2$

$-2y = (x+3)^2 + 20$

$y = \frac{-1}{2}(x+3)^2 - 10$

vertex: $(-3, -10)$

aos: $x = -3$

y-intercept: $\left(0, \frac{-29}{2}\right)$

x-intercepts: none

Use the information provided to write the standard form equation of each ellipse.

41) $9x^2 + 16y^2 + 180x + 64y + 388 = 0$

$9(x^2 + 20x + \underline{100} - 100) + 16(y^2 + 4y + \underline{4} - 4) + 388 = 0$

$9(x+10)^2 \underline{-900} + 16(y+2)^2 \underline{-64} = -388$

$+900$ $+64$

$9(x+10)^2 + 16(y+2)^2 = 576$

$$\frac{(x+10)^2}{64} + \frac{(y+2)^2}{36} = 1$$

42) Vertices: $(-2, 21), (-2, -5)$

Foci: $(-2, 8 + 3\sqrt{17}), (-2, 8 - 3\sqrt{17})$

major = y axis

center $\left(\frac{-2+-2}{2}, \frac{21+-5}{2}\right) = (-2, 8)$

$a = 13$ $c = 3\sqrt{17}$

$a^2 - c^2 = b^2$

$169 - 9(17) = b^2$

$169 - 153 = b^2$

$16 = b^2$

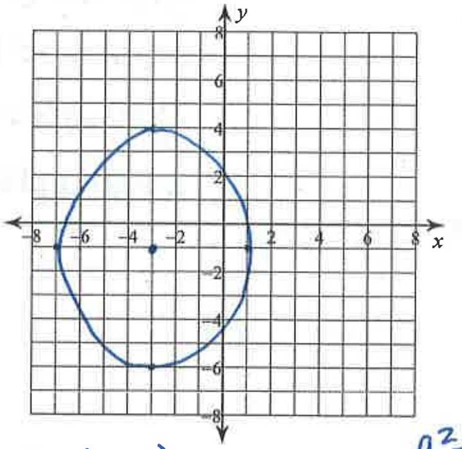
$b = 4$

$$\frac{(x+2)^2}{16} + \frac{(y-8)^2}{169} = 1$$

Identify the center, vertices, and foci of each. Then sketch the graph.

43) $\frac{(x+3)^2}{16}+\frac{(y+1)^2}{25}=1$

major= y-axis



center (-3,-1)

major vertices (-3,4)(-3,-6)

minor vertices (-7,-1)(1,-1)

a=5 b=4

$a^2-c^2=b^2$

$25-c^2=16$

$-c^2=-9$

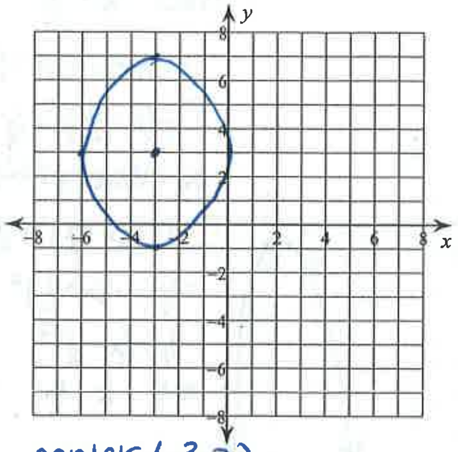
$c=3$

Foci

(-3,2) (-3,-4)

44) $\frac{(x+3)^2}{9}+\frac{(y-3)^2}{16}=1$

major = y-axis



center (-3,3)

major vertices (-3,7)(-3,-1)

minor vertices (-6,3)(0,3)

a=4 b=3

$a^2-c^2=b^2$

$16-c^2=9$

$c^2=7$

$c=\sqrt{7}$

Foci

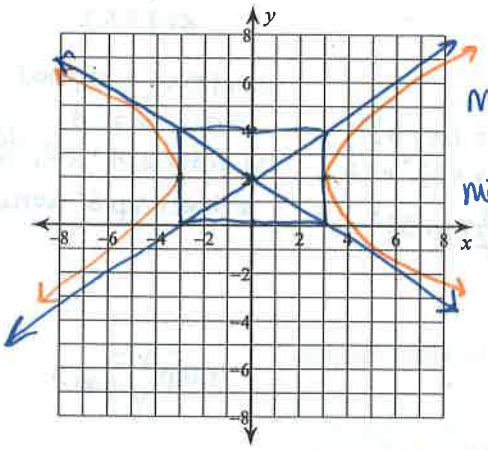
$(-3,3+\sqrt{7})(-3,3-\sqrt{7})$

Identify the vertices, foci, and asymptotes of each. Then sketch the graph.

45) $\frac{x^2}{9}-\frac{(y-2)^2}{4}=1$

major=x-axis

a=3 b=2



Major vertices:

(-3,2) (3,2)

minor vertices:

(0,4)(0,0)

Center (0,2)

Foci $c^2-a^2=b^2$

$c^2-9=4$

$c^2=13$

$c=\sqrt{13}$

$(\sqrt{13},2)(-\sqrt{13},2)$

asymptotes:

$y=2\pm\frac{2}{3}(x-0)^2$

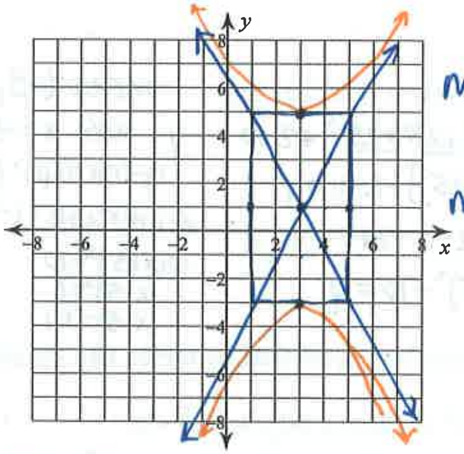
$y=\frac{2}{3}x+2$

$y=\frac{-2}{3}x+2$

46) $\frac{(y-1)^2}{16}-\frac{(x-3)^2}{4}=1$

major = y-axis

a=2 b=4



Major vertices:

(3,5)(3,-3)

minor vertices:

(5,1)(1,1)

center (3,1)

$c^2-4=16$

$c^2=20$

$c=2\sqrt{5}$

Foci

$(3,1+2\sqrt{5})(3,1-2\sqrt{5})$

asymptotes:

$y=1\pm\frac{4}{2}(x-3)$

$y=2x-5$

$y=-2x+7$

Use the information provided to write the standard form equation of each hyperbola.

47) $-x^2 + 4y^2 - 20x + 64y - 40 = 0$

$-(x^2+20x+\underline{100}-100)+4(y^2+16y+\underline{64}-\underline{64})=40$

$-(x+10)^2+100+4(y+8)^2-256=40$ $\quad \begin{matrix}-100\\ \underline{+256}\end{matrix}$

$4(y+8)^2-(x+10)^2=196$

$\dfrac{(y+8)^2}{49}-\dfrac{(x+10)^2}{196}=1$

48) Vertices: $(-7, 9), (-7, -15)$
Foci: $(-7, 10), (-7, -16)$

major = y-axis

center $\left(\frac{-7-7}{2}, \frac{9-15}{2}\right) = (-7,-3)$

$a=12 \quad c=13$

$c^2-a^2=b^2$

$169-144=b^2$

$25=b^2$

$5=b$

$\dfrac{(y+3)^2}{144}-\dfrac{(x+7)^2}{25}=1$

Find the domain and range of the given function.

49) $Y(t) = 5t^2 + 3t - 8$

D: $\mathbb{R}$ $\quad$ R: $[-8,\infty)$

50) $f(z) = -\sqrt{7z + 1}$

D: $[-\frac{1}{7},\infty)$

$7z+1\geq 0$

$7z\geq -1$

$z\geq \frac{-1}{7}$

R: $(-\infty,0]$

$\uparrow -\underbrace{\sqrt{7z+1}}_{0 \text{ or } +}$

multiplied by a negative

51) $f(z) = \sqrt{z + 3} + \sqrt{z^2 + 5}$

D: $[-3,\infty)$

$z+3\geq 0 \quad z^2+5\geq 0$

$z\geq -3$ $\quad$ Factor doesn't work
Quadratic doesn't work

$z^2\geq -5$

always $\mathbb{R}$

R: $[\sqrt{5},\infty)$

$\sqrt{z+3}$ and $\sqrt{z^2+5}$

0 or +

special case!

$\uparrow$

$(0 \text{ or } +)+5$

$\sqrt{5}$

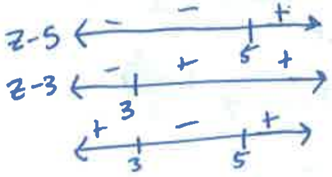
Domain only!

52) $h(z) = \sqrt{z^2 - 8z + 15}$

D: $(-\infty,3]\cup[5,\infty)$

$z^2-8z+15\geq 0$

$(z-5)(z+3)\geq 0$



53) $M(x) = |x - 1| + 27$

D: $\mathbb{R}$ $\quad$ R: $[27,\infty)$

$\underbrace{|x-1|}_{0 \text{ or } +}+27$

54) $f(x) = 3 + x$

D: $\mathbb{R}$ $\quad$ R: $\mathbb{R}$

Find the domain of the given function.

55) $f(w) = \dfrac{w^3 - 3w + 1}{2w - 8}$

$2w - 8 = 0$

$2w = 8$

$w \neq 4$

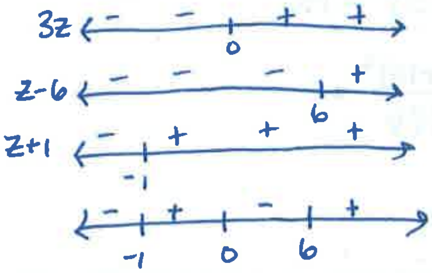
$(-\infty, 4) \cup [4, \infty)$

56) $R(z) = \dfrac{5}{3z^3 - 15z^2 - 18z}$

$z(3z^2 - 15z - 18) > 0$

$3z(z^2 - 5z - 6) > 0$

$3z(z-6)(z+1) > 0$



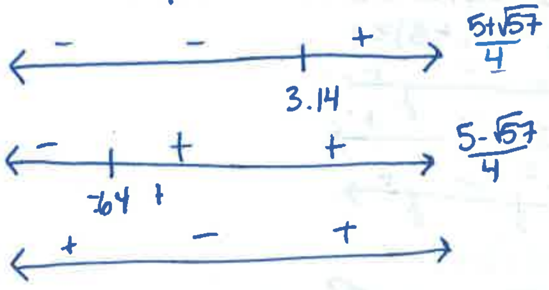
$(-1, 0) \cup (6, \infty)$

57) $g(t) = \sqrt{2x^2 - 5x - 4}$

$2x^2 - 5x - 4 \geq 0$

$\dfrac{5 \pm \sqrt{25 - 4(2)(-4)}}{2(2)} \geq 0$

$\dfrac{5 \pm \sqrt{57}}{4} \geq 0$



$\left(-\infty, \dfrac{5 - \sqrt{57}}{4}\right] \cup \left[\dfrac{5 + \sqrt{57}}{4}, \infty\right)$

58) $h(y) = \sqrt{y + 3} - \dfrac{1}{\sqrt{5 - y}}$

$y + 3 \geq 0$

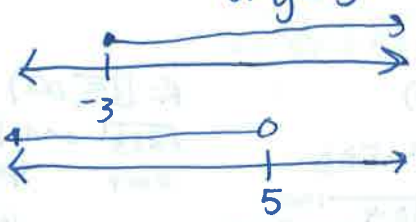
$y \geq -3$

AND

$5 - y > 0$

$5 > y$

or $y < 5$



overlap

$[-3, 5)$