Calculus

Name Key

Limits - Review

Date________

Evaluate each limit.

1) $\lim_{x\to 3} \frac{x^2-9}{x-3} = \frac{0}{0}$ $\quad \frac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}}$

$\lim_{x\to 3} x+3 = 6$

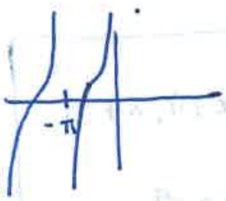
2) $\lim_{x\to 2} \frac{x-2}{x^2-4} = \frac{0}{0}$ $\quad \frac{\cancel{(x-2)}}{\cancel{(x-2)}(x+2)}$

$\lim_{x\to 2} \frac{1}{x+2} = \frac{1}{4}$

3) $\lim_{x\to -\pi} -2\cot(x) = \frac{-2}{\tan(-\pi)} = \frac{-2}{0}$

asymptote

DNE



4) $\lim_{x\to 0} \frac{x\sec 4x}{\sin 4x} = x \cdot \frac{1}{\cos 4x} \cdot \frac{1}{\sin 4x} \cdot \left(\frac{4}{4}\right)$

$= \frac{1}{4\cos 4x} = \frac{1}{4(1)} = \frac{1}{4}$

5) $\lim_{x\to -\infty} \frac{2x^2}{x^2-4} = \frac{-\infty}{-\infty}$

$= \frac{2\cancel{x^2}}{\cancel{x^2}\left(1-\frac{4}{x^2}\right)} = \frac{2}{1} = 2$

6) $\lim_{x\to -\infty} \left(-e^{-2x}+1\right) = -(+\infty)+1 = -\infty$

7) $\lim_{x\to \infty} \left(-x^3+3x^2-3\right) = -x^3\left(1-\frac{3}{x}+\frac{3}{x^3}\right)$

$= -x^3 = -\infty$

8) $\lim_{x\to 2} \frac{\left(\sqrt{x+2}-2\right)}{x-2} \cdot \frac{(\sqrt{x+2}+2)}{(\sqrt{x+2}+2)} = \frac{x+2-4}{(x-2)(\sqrt{x+2}+2)}$

$\lim_{x\to 2} = \frac{\cancel{(x-2)}}{\cancel{(x-2)}(\sqrt{x+2}+2)} = \frac{1}{\sqrt{x+2}+2} = \frac{1}{4}$

9) $\lim_{x\to 0} \frac{\cos(2x)-\cos(4x)}{x} = -2\sin\left(\frac{2x+4x}{2}\right)\sin\left(\frac{2x-4x}{2}\right)$

$\lim_{x\to 0} -2\sin(3x)\sin(-x)\cdot\frac{1}{x}$

$= 2\sin(3x)\cdot \boxed{\frac{\sin(-x)}{-x}}$

$= 2\sin(0)\cdot 1$

$= 2\cdot 0\cdot 1 = \boxed{0}$

10) $\lim_{x\to 0} \frac{x}{\frac{1}{-1+x}+1} = \frac{x}{\frac{1}{-1+x}+\frac{-1+x}{-1+x}}$

$= \frac{x}{\frac{1-1+x}{-1+x}} = \frac{x}{\frac{x}{-1+x}}$

$= x \cdot \frac{-1+x}{x} = -1+x = -1+0 = -1$

Find the interval of continuity of the composition of functions.

11) $f(x) = \sqrt{x^2 - 6x - 16}$

① $\sqrt{u}$ D: $[0, \infty)$ aka $x \geq 0$

② $x^2 - 6x - 16$ D: $(-\infty, \infty)$

$x^2 - 6x - 16 \geq 0$

$(x-8)(x+2) \geq 0$

$x-8$: $-$ $-$ | 8 | $+$

$x+2$: $-$ | -2 | $+$ $+$

overall: $+$ | -2 | $-$ | 8 | $+$

positive

$\boxed{(-\infty, -2) \cup (8, \infty)}$

Note all points of discontinuity for the function. Remove the discontinuity when possible by creating a piecewise function. If discontinuity cannot be removed, explain why.

12) $f(x) = \dfrac{x^4 + x^3 - 81x^2 - 81x}{x^2 + 10x + 9} = \dfrac{x(x^3 + x^2 - 81x - 81)}{(x+9)(x+1)} = \dfrac{x[x^2(x+1) - 81(x+1)]}{(x+9)(x+1)}$

$= \dfrac{x(x^2 - 81)(x+1)}{(x+9)(x+1)} = \dfrac{x\cancel{(x+9)}(x-9)\cancel{(x+1)}}{\cancel{(x+9)}\cancel{(x+1)}}$

$\lim\limits_{x \to -9} x(x-9) = -9(-18) = 162$

$\lim\limits_{x \to -1} x(x-9) = -1(-10) = 10$

$$\boxed{f(x) = \begin{cases} \dfrac{x^4 + x^3 - 81x^2 - 81x}{x^2 + 10x + 9} & x \neq -9,\ x \neq -1 \\ 162 & x = -9 \\ 10 & x = -1 \end{cases}}$$

Identify all vertical, horizontal, and slant asymptotes as well as any holes. Give the equations of each asymptote.

13) $f(x) = \dfrac{x^3 + 3x^2 - 9x - 27}{x^2 + 2x - 3} = \dfrac{x^2(x+3) - 9(x+3)}{x^2 + 2x - 3} = \dfrac{(x^2 - 9)(x+3)}{x^2 + 2x - 3} = \dfrac{(x+3)(x-3)\cancel{(x+3)}}{\cancel{(x+3)}(x-1)}$

VA	$x = 1$
hole	$x = -3$
HA	no HA
Slant	$y = x + 1$

(power of numerator > denominator)

$$\begin{array}{r} x + 1 - \dfrac{8x+24}{x^2+2x-3} \\ x^2 + 2x - 3 \overline{\big)\, x^3 + 3x^2 - 9x - 27} \\ -\ x^3 + {}^-2x^2 + 3x \\ \hline x^2 - 6x - 27 \\ -\ x^2 + {}^-2x + 3 \\ \hline -8x - 24 \end{array}$$