69-74, 77-78

$$C(x) = 4000 - 32x + 0.08x^2 + 0.00006x^3$$

and the demand function for the widgets is given by,

$$p(x) = 250 + 0.02x - 0.001x^2$$

What is the marginal cost, marginal revenue and marginal profit when $x = 200$ and $x = 400$? What do these numbers tell you about the cost, revenue and profit?

Integrals

Introduction

Here are a set of practice problems for the Integrals chapter of my Calculus I notes. If you are viewing the pdf version of this document (as opposed to viewing it on the web) this document contains only the problems themselves and no solutions are included in this document. Solutions can be found in a number of places on the site.

16. If you'd like a pdf document containing the solutions go to the note page for the section you'd like solutions for and select the download solutions link from there. Or,

17. Go to the download page for the site http://tutorial.math.lamar.edu/download.aspx and select the section you'd like solutions for and a link will be provided there.

18. If you'd like to view the solutions on the web or solutions to an individual problem you can go to the problem set web page, select the problem you want the solution for. At this point I do not provide pdf versions of individual solutions, but for a particular problem you can select "Printable View" from the "Solution Pane Options" to get a printable version.

Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section.

Here is a list of topics in this chapter that have practice problems written for them.

Definition of the Definite Integral
Computing Definite Integrals
Substitution Rule for Definite Integrals

Indefinite Integrals

1. Evaluate each of the following indefinite integrals.

   (a) $\int 6x^5 - 18x^2 + 7\,dx$ $x^6 - 6x^3 + 7x + C$

   (b) $\int \underbrace{6x^5\,dx} - 18x^2 + 7$ $\underbrace{x^6 + C}_{\text{Integrate here ONLY}} - 18x^2 + 7$

2. Evaluate each of the following indefinite integrals.

   (a) $\int 40x^3 + 12x^2 - 9x + 14\,dx$ $10x^4 + 4x^3 - \frac{9}{2}x^2 + 14x + C$

   (b) $\int \underline{40x^3 + 12x^2 - 9x\,dx} + 14$ $10x^4 + 4x^3 - \frac{9}{2}x^2 + C + 14$

   (c) $\int \underline{40x^3 + 12x^2\,dx} - 9x + 14$ $10x^4 + 4x^3 + C - 9x + 14$

For problems 3 – 5 evaluate the indefinite integral.

3. $\int 12t^7 - t^2 - t + 3\,dt$ $\frac{3}{2}t^8 - \frac{1}{3}t^3 - \frac{1}{2}t^2 + 3t + C$

4. $\int 10w^4 + 9w^3 + 7w\,dw$ $2w^5 + \frac{9}{4}w^4 + \frac{7}{2}w^2 + C$

5. $\int z^6 + 4z^4 - z^2\,dz$ $\frac{1}{7}z^7 + \frac{4}{5}z^5 - \frac{1}{3}z^3 + C$

6. Determine $f(x)$ given that $f'(x) = 6x^8 - 20x^4 + x^2 + 9$. $f(x) = \frac{2}{3}x^9 - 4x^5 + \frac{1}{3}x^3 + 9x + C$

7. Determine $h(t)$ given that $h'(t) = t^4 - t^3 + t^2 + t - 1$. $h(t) = \frac{1}{5}t^5 - \frac{1}{4}t^4 + \frac{1}{3}t^3 + \frac{1}{2}t^2 - t + C$

Computing Indefinite Integrals

For problems 1 – 21 evaluate the given integral.

1. $\int 4x^6 - 2x^3 + 7x - 4\,dx$ $\frac{4}{7}x^7 - \frac{1}{2}x^4 + \frac{7}{2}x^2 - 4x + C$

2. $\int z^7 - 48z^{11} - 5z^{16}\, dz$ $\quad \frac{1}{8}z^8 - 4z^{12} - \frac{5}{17}z^{17} + c$

3. $\int 10t^{-3} + 12t^{-9} + 4t^3\, dt$ $\quad -5t^{-2} - \frac{3}{2}t^{-8} + \ t^4 + c$

4. $\int w^{-2} + 10w^{-5} - 8\, dw$ $\quad -w^{-1} - \frac{5}{2}w^{-4} - 8w + c$

5. $\int 12\, dy$ $\quad 12y + c$

6. $\int \sqrt[3]{w} + 10\sqrt[5]{w^3}\, dw = \int w^{1/3} + 10w^{3/5}\, dw$ $\quad \boxed{\frac{3}{4}w^{4/3} + \frac{25}{4}w^{8/5} + c}$

7. $\int \sqrt{x^7} - 7\sqrt[6]{x^5} + 17\sqrt[3]{x^{10}}\, dx = \int x^{7/2} - 7x^{5/6} + 17x^{10/3}\, dx$ $\quad \boxed{\frac{2}{9}x^{9/2} - \frac{42}{11}x^{11/6} + \frac{51}{13}x^{13/3} + c}$

8. $\int \frac{4}{x^2} + 2 - \frac{1}{8x^3}\, dx = \int 4x^{-2} + 2 - \frac{1}{8}x^{-3}\, dx$ $\quad \boxed{-4x^{-1} + 2x + \frac{1}{16}x^{-2} + c}$

9. $\int \frac{7}{3y^6} + \frac{1}{y^{10}} - \frac{2}{\sqrt[3]{y^4}}\, dy = \int \frac{7}{3}y^{-6} + y^{-10} - 2y^{-4/3}\, dy$ $\quad \boxed{-\frac{7}{15}y^{-5} - \frac{1}{9}y^{-9} + 6y^{-1/3} + c}$

10. $\int (t^2 - 1)(4 + 3t)\, dt = \int 3t^3 + 4t^2 - 3t - 4\, dt$ $\quad \boxed{\frac{3}{4}t^4 + \frac{4}{3}t^3 - \frac{3}{2}t^2 - 4t + c}$

add exponents

11. $\int \underbrace{\sqrt{z}}_{z^{1/2}}\left(z^2 - \underbrace{\frac{1}{4z}}_{\frac{1}{4}z^{-1}}\right) dz = \int z^{5/2} - \frac{1}{4}z^{-1/2}\, dz$ $\quad \boxed{\frac{2}{7}z^{7/2} - \frac{1}{2}z^{1/2} + c}$

12. $\int \frac{z^8 - 6z^5 + 4z^3 - 2}{z^4}\, dz = \int z^4 - 6z + \underbrace{4z^{-1}}_{\ln \text{ rule } \frac{4}{z}} - 2z^{-4}\, dz$ $\quad \boxed{\frac{1}{5}z^5 - 3z^2 + 4\ln|z| + \frac{2}{3}z^{-3} + c}$

subtract exponents

13. $\int \frac{x^4 - \sqrt[3]{x}}{6\sqrt{x}}\, dx = \int \frac{1}{6}x^{7/2} - \frac{1}{6}x^{-1/6}\, dx$ $\quad \boxed{\frac{1}{27}x^{9/2} - \frac{1}{5}x^{5/6} + c}$

($\sqrt[3]{x} = x^{1/3}$, $6\sqrt{x} = 6x^{1/2}$)

14. $\int \sin(x) + 10\csc^2(x)\, dx$ $\quad \boxed{-\cos(x) - 10\cot(x) + c}$

15. $\int 2\cos(w) - \sec(w)\tan(w)\, dw$ $\quad \boxed{2\sin(x) - \sec(w) + c}$

16. $\int 12 + \csc(\theta)[\sin(\theta) + \csc(\theta)]\, d\theta = \int 12 + \underbrace{\csc\theta \sin\theta}_{\frac{1}{\sin\theta}\cdot\frac{\sin\theta}{1} = 1} + \csc^2\theta\, d\theta = \int 13 + \csc^2\theta\, d\theta$ $\quad \boxed{13\theta - \cot(\theta) + c}$

$-\frac{1}{6}z^{-1}$ ← ln rule

17. $\int 4\mathbf{e}^z + 15 - \frac{1}{6z}\,dz$ $\boxed{4e^z + 15z - \frac{1}{6}\ln|z| + C}$

18. $\int t^3 - \frac{\mathbf{e}^{-t} - 4}{\mathbf{e}^{-t}}\,dt = \int t^3 - 1 + 4e^t\,dt$ $\boxed{\frac{1}{4}t^4 - t + 4e^t + C}$

↙ ln rule

19. $\int \frac{6}{w^3} - \frac{2}{w}\,dw = \int 6w^{-3} - \frac{2}{w}\,dw$ $\boxed{-3w^{-2} - 2\ln|w| + C}$

20. $\int \frac{1}{1+x^2} + \frac{12}{\sqrt{1-x^2}}\,dx$ $\boxed{\tan^{-1}(x) + 12\sin^{-1}(x) + C}$

21. $\int 6\cos(z) + \frac{4}{\sqrt{1-z^2}}\,dz$ $\boxed{6\sin(z) + 4\sin^{-1}(z) + C}$ or $6\sin(z) - 4\cos^{-1}(z) + C$

22. Determine $f(x)$ given that $f'(x) = 12x^2 - 4x$ and $f(-3) = 17$.

$\boxed{f(x) = 4x^3 - 2x^2 + 143}$

$f(-3) = 4(-3)^3 - 2(-3)^2 + C = 17$

$-108 - 18 + C = 17$

$-126 + C = 17$ $C = 143$

23. Determine $g(z)$ given that $g'(z) = 3z^3 + \frac{7}{2\sqrt{z}} - \mathbf{e}^z$ and $g(1) = 15 - \mathbf{e}$.

$3z^3 + \frac{7}{2}z^{-1/2} - e^z$

$\boxed{g(z) = \frac{3}{4}z^4 + 7z^{1/2} - e^z + 29/4}$

$g(1) = \frac{3}{4}(1)^4 + 7(1)^{1/2} - e^1 + C = 15 - e$

$3/4 + 7 - e + C = 15 - e$

$\frac{31}{4} - e + C = 15 - e$

$C = 29/4$

Challenge

24. Determine $h(t)$ given that $h''(t) = 24t^2 - 48t + 2$, $h(1) = -9$ and $h(-2) = -4$.

$h'(t) = 8t^3 - 24t^2 + 2t + C$

$h(t) = 2t^4 - 8t^3 + t^2 + Ct + d$

$2(1)^4 - 8(1)^3 + (1)^2 + C + d = -9$

$-5 + C + d = -9$

$C + d = -4$ x2

$2(-2)^4 - 8(-2)^3 + (-2)^2 - 2C + d = -4$

$100 - 2C + d = -4$

$-2C + d = -104$

$2C + 2d = -8$

$3d = -112$ $d = \frac{-112}{3}$

$C + \frac{-112}{3} = -4$

$C = \frac{100}{3}$

Final Answer $h(t) = 2t^4 - 8t^3 + t^2 + \frac{100}{3}t - \frac{112}{3}$

Substitution Rule for Indefinite Integrals

For problems 1 – 16 evaluate the given integral.

1. $\int (8x-12)\underbrace{(4x^2 - 12x)}_{u}{}^4\,dx$ $u = 4x^2 - 12x$ $du = 8x - 12dx$ $\int u^4\,du$ $\frac{1}{5}u^5 + C =$ $\boxed{\frac{1}{5}(4x^2 - 12x)^5 + C}$

$\frac{-1}{4}$ -4

2. $\int 3t^{-4}(2 + 4t^{-3})^{-7}\,dt$ $u = 2 + 4t^{-3}$ $du = -12t^{-4}\,dt$ $\frac{-1}{4}\int u^{-7}\,du$ $\frac{1}{24}u^{-6} + C =$ $\boxed{\frac{1}{24}(2 + 4t^{-3})^{-6} + C}$

$-\frac{1}{2}$ -2

3. $\int (3-4w)(4w^2 - 6w + 7)^{10}\,dw$ $u = 4w^2 - 6w + 7$ $du = 8w - 6\,dw$ $= -2(3-4w)\,dw$ $\frac{-1}{2}\int u^{10}\,du$ $\frac{-1}{22}u^{11} + C =$ $\boxed{\frac{-1}{22}(4w^2 - 6w + 7)^{11} + C}$

$\frac{5}{2}$ $2/5$

4. $\int 5(z-4)\sqrt[3]{z^2 - 8z}\,dz$ $u = z^2 - 8z$ $du = 2z - 8\,dz$ $= 2(z-4)\,dz$ $\frac{5}{2}\int u^{1/3}\,du$ $\frac{15}{8}u^{4/3} + C =$ $\boxed{\frac{15}{8}(z^2 - 8z)^{4/3} + C}$

5 $1/6$

5. $\int 90x^2\sin(2 + 6x^3)\,dx$ $u = 2 + 6x^3$ $du = 18x^2\,dx$ $5\int \sin u\,du$ $-5\cos u + C =$ $\boxed{-5\cos(2 + 6x^3) + C}$

6. $\int \sec(1-z)\tan(1-z)\,dz$ $\quad u = 1-z$, $du = -1\,dz$ $\quad -\int \sec(u)\tan(u)\,du \quad -\sec(u)+c = \boxed{-\sec(1-z)+c}$

7. $\int (15t^{-2}-5t)\cos(6t^{-1}+t^2)\,dt$ $\quad u = 6t^{-1}+t^2$, $du = -6t^{-2}+2t\,dt = -\frac{2}{5}(15t^{-2}-5t)dt$ $\quad -\frac{5}{2}\int \cos u\,du \quad -\frac{5}{2}\sin u + c = -\frac{5}{2}\sin(6t^{-1}+t^2)+c$

8. $\int (7y-2y^3)e^{y^4-7y^2}\,dy$ $\quad u = y^4-7y^2$, $du = 4y^3-14y\,dy = -2(7y-2y^3)dy$ $\quad -\frac{1}{2}\int e^u\,du \quad -\frac{1}{2}e^u + c = \boxed{-\frac{1}{2}e^{y^4-7y^2}+c}$

9. $\int \frac{4w+3}{4w^2+6w-1}\,dw$ $\quad u = 4w^2+6w-1$, $du = 8w+6\,dw = 2(4w+3)dw$ $\quad \frac{1}{2}\int \frac{1}{u}\,du$ ln rule! $\quad \frac{1}{2}\ln|u|+c = \boxed{\frac{1}{2}\ln|4w^2+6w-1|+c}$

10. $\int (\cos(3t)-t^2)(\sin(3t)-t^3)^5\,dt$ $\quad u = \sin(3t)-t^3$, $du = 3\cos(3t)-3t^2\,dt = 3(\cos(3t)-t^2)dt$ $\quad \frac{1}{3}\int u^5\,du \quad \frac{1}{18}u^6 + c = \boxed{\frac{1}{18}(\sin(3t)-t^3)^6+c}$

11. $\int 4\left(\frac{1}{z}-e^{-z}\right)\cos(e^{-z}+\ln z)\,dz$ $\quad u = e^{-z}+\ln z$, $du = -e^{-z}+\frac{1}{z}\,dz$ $\quad 4\int \cos u\,du \quad 4\sin u + c = \boxed{4\sin(e^{-z}+\ln z)+c}$

12. $\int \sec^2(v)e^{1+\tan(v)}\,dv$ $\quad u = 1+\tan(v)$, $du = \sec^2(v)\,dv$ $\quad \int e^u\,du \quad e^u + c = \boxed{e^{1+\tan(v)}+c}$

13. $\int 10\sin(2x)\cos(2x)\sqrt{\cos^2(2x)-5}\,dx$ $\quad u = (\cos(2x))^2-5$, $du = 2\cos(2x)\cdot -\sin(2x)\cdot 2\,dx = -4\sin(2x)\cos(2x)\,dx$ $\quad -\frac{5}{2}\int u^{1/2}du \quad -\frac{5}{3}u^{3/2}+c = \boxed{-\frac{5}{3}(\cos^2(2x)-5)^{3/2}+c}$

14. $\int \frac{\csc(x)\cot(x)}{2-\csc(x)}\,dx$ $\quad u = 2-\csc(x)$, $du = \csc(x)\cot(x)\,dx$ $\quad \int \frac{1}{u}\,du$ ln rule $\quad \ln|u|+c = \boxed{\ln|2-\csc(x)|+c}$

15. $\int \frac{6}{7+y^2}\,dy$ $\quad 7(1+\frac{1}{7}y^2)$ $\quad \cancel{u = 7+y^2}$, $\cancel{du = 2y\,dy}$ $\quad$ looks like $\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$

$\frac{6}{7}\int \frac{1}{1+\frac{1}{7}y^2}\,dy = \frac{6}{7}\int \frac{1}{1+\left(\frac{1}{\sqrt{7}}y\right)^2}\,dy$ $\quad u = \frac{1}{\sqrt{7}}y$, $du = \frac{1}{\sqrt{7}}dy$ $\quad \frac{6\sqrt{7}}{7}\int \frac{1}{1+u^2}\,du$

$\frac{6\sqrt{7}}{7}\tan^{-1}(u)+c = \boxed{\frac{6\sqrt{7}}{7}\tan^{-1}\left(\frac{1}{\sqrt{7}}y\right)+c}$

16. $\int \frac{1}{\sqrt{4-9w^2}}\,dw$ $\quad \sqrt{4(1-\frac{9}{4}w^2)}$ comes out as $\sqrt{4}=2$ $\quad \cancel{u = 4-9w^2}$, $\cancel{du = -18w\,dw}$ $\quad$ looks like $\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$

$\frac{1}{2}\int \frac{1}{\sqrt{1-\frac{9}{4}w^2}}\,dw = \frac{1}{2}\int \frac{1}{\sqrt{1-\left(\frac{3}{2}w\right)^2}}\,dw$ $\quad u = \frac{3}{2}w$, $du = \frac{3}{2}dw$

$\frac{1}{3}\int \frac{1}{\sqrt{1-u^2}}\,du \quad \frac{1}{3}\sin^{-1}(u)+c = \boxed{\frac{1}{3}\sin^{-1}\left(\frac{3}{2}w\right)+c}$

17. Evaluate each of the following integrals.

(a) $\int \frac{3x}{1+9x^2}\,dx$ $\quad u = 1+9x^2$, $du = 18x\,dx$ $\quad \frac{1}{6}\int \frac{1}{u}\,du \quad \frac{1}{6}\ln|u|+c = \boxed{\frac{1}{6}\ln|1+9x^2|+c}$

(b) $\int \frac{3x}{(1+9x^2)^4}\,dx$ $\quad u = 1+9x^2$, $du = 18x\,dx$ $\quad \frac{1}{6}\int \frac{1}{u^4}\,du = \frac{1}{6}\int u^{-4}\,du \quad -\frac{1}{18}u^{-3}+c = \boxed{-\frac{1}{18}(1+9x^2)^{-3}+c}$

(c) $\int \frac{3}{1+9x^2}\,dx$ $\quad \cancel{u = 1+9x^2}$, $\cancel{du = 18x\,dx}$ $\quad$ looks like $\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$

$3\int \frac{1}{1+9x^2}\,dx = \frac{3}{3}\int \frac{1}{1+(3x)^2}\,dx$ $\quad u = 3x$, $du = 3\,dx$ $\quad \int \frac{1}{1+u^2}\,du$

$\tan^{-1}(u)+c = \boxed{\tan^{-1}(3x)+c}$

More Substitution Rule

Evaluate each of the following integrals.

1. $\int 4\sqrt{5+9t} + 12(5+9t)^7\, dt$

$u=5+9t$, $du=9\,dt$

$\frac{4}{9}\int u^{1/2}\,du + \frac{4}{3}\int u^7\,du$

$\frac{4}{9}\cdot\frac{2}{3}u^{3/2} + \frac{4}{3}\cdot\frac{1}{8}u^8 + c = \boxed{\frac{8}{27}(5+9t)^{3/2} + \frac{1}{6}(5+9t)^8 + c}$

2. $\int 7x^3\cos(2+x^4) - 8x^3 e^{2+x^4}\, dx$

$u=2+x^4$, $du=4x^3dx$

$\frac{7}{4}\int \cos u\,du - 2\int e^u du$

$\frac{7}{4}\sin u - 2e^u + c = \boxed{\frac{7}{4}\sin(2+x^4) - 2e^{2+x^4} + c}$

3. $\int \frac{6e^{7w}}{(1-8e^{7w})^3} + \frac{14e^{7w}}{1-8e^{7w}}\, dw$

$u=1-8e^{7w}$, $du=-56e^{7w}dw$

$-\frac{3}{28}\int u^{-3}du - \frac{1}{4}\int \frac{1}{u}du$

$-\frac{3}{28}\cdot\frac{1}{-2}u^{-2} - \frac{1}{4}\ln|u| + c = \boxed{\frac{3}{56(1-8e^{7w})^2} - \frac{1}{4}\ln|1-8e^{7w}| + c}$

4. $\int x^4 - 7x^5\cos(2x^6+3)\, dx$

$u=2x^6+3$, $du=12x^5dx$

$\int x^4dx - \frac{7}{12}\int \cos u\,du$

$\frac{1}{5}x^5 - \frac{7}{12}\sin u + c = \boxed{\frac{1}{5}x^5 - \frac{7}{12}\sin(2x^6+3) + c}$

5. $\int e^z + \frac{4\sin(8z)}{1+9\cos(8z)}\, dz$

$u=1+9\cos(8z)$, $du=-72\sin(8z)dz$

$\int e^z - \frac{1}{18}\int\frac{1}{u}du$

$e^z - \frac{1}{18}\ln|u| + c = \boxed{e^z - \frac{1}{18}\ln|1+9\cos(8z)| + c}$

6. $\int 20e^{2-8w}\sqrt{1+e^{2-8w}} + 7w^3 - 6\sqrt[3]{w}\, dw$

$u=1+e^{2-8w}$, $du=-8e^{2-8w}dw$

$-\frac{5}{2}\int u^{1/2}du + 7\int w^3dw - 6\int w^{1/3}dw$

$-\frac{5}{2}\cdot\frac{2}{3}u^{3/2} + 7\cdot\frac{1}{4}w^4 - 6\cdot\frac{3}{4}w^{4/3} + c = \boxed{-\frac{5}{3}(1+e^{2-8w})^{3/2} + \frac{7}{4}w^4 - \frac{9}{2}w^{4/3} + c}$

7. $\int (4+7t)^3 - 9t\sqrt[4]{5t^2+3}\, dt$

$u=4+7t$, $du=7\,dt$; $u=5t^2+3$, $du=10t\,dt$

$\frac{1}{7}\int u^3du - \frac{9}{10}\int u^{1/4}du$

$\frac{1}{7}\cdot\frac{1}{4}u^4 - \frac{9}{10}\cdot\frac{4}{5}u^{5/4} + c$

$= \boxed{\frac{1}{28}(4+7t)^4 - \frac{18}{25}(5t^2+3)^{5/4} + c}$

8. $\int \frac{6x-x^2}{x^3-9x^2+8} - \csc^2\left(\frac{3x}{2}\right) dx$

$u=x^3-9x^2+8$, $du=3x^2-18x\,dx$; $u=\frac{3x}{2}$, $du=\frac{3}{2}dx$

$-\frac{1}{3}\int\frac{1}{u}du - \frac{2}{3}\int\csc^2 u\,du$

$-\frac{1}{3}\ln|u| + \frac{2}{3}\cot u + c$

$\boxed{-\frac{1}{3}\ln|x^3-9x^2+8| + \frac{2}{3}\cot\left(\frac{3x}{2}\right) + c}$

9. $\int 7(3y+2)(4y+3y^2)^3 + \sin(3+8y)\, dy$

10. $\int \sec^2(2t)\left[9+7\tan(2t)-\tan^2(2t)\right] dt$

see separate sheet

11. $\int \frac{8-w}{4w^2+9}\, dw$

12. $\int \frac{7x+2}{\sqrt{1-25x^2}}\, dx$

More Substitution Rule

9. $\frac{1}{2}\cdot 7\int \cancel{7}(3y+2)\,(\underbrace{4y+3y^2}})^3 + \sin(\underbrace{3+8y})\,dy$

$u = 4y+3y^2$ $\qquad$ $u = 3+8y$

$du = 4+6y\,dy$ $\qquad$ $du = 8\,dy$

$\frac{7}{2}\int u^3\,du + \frac{1}{8}\int \sin u\,du$

$\frac{7}{2}\cdot\frac{1}{4}u^4 + -\frac{1}{8}\cos u + C = \boxed{\frac{7}{8}(4y+3y^2)^4 - \frac{1}{8}\cos(3+8y) + C}$

10. $\int \sec^2(2t)\left[9+7\tan(2t)-\tan^2(2t)\right]dt$

$\frac{1}{2}\cdot 9\int \cancel{9}\sec^2(2t)\,dt + 7\int \cancel{7}\sec^2(2t)\underbrace{\tan(2t)}\,dt - \frac{1}{2}\int \sec^2(2t)\underbrace{\tan^2(2t)}\,dt$

$u = 2t$ $\qquad$ $u = \tan(2t)$ chain rule $\qquad$ $u = \tan(2t)$

$du = 2\,dt$ $\qquad$ $du = 2\sec^2(2t)\,dt$ $\qquad$ $du = 2\sec^2(2t)\,dt$

$\frac{9}{2}\int \sec^2(u)\,du + \frac{7}{2}\int u\,du - \frac{1}{2}\int u^2\,du$

$\frac{9}{2}\tan(u) + \frac{7}{2}\cdot\frac{1}{2}u^2 - \frac{1}{2}\cdot\frac{1}{3}u^3 + C = \boxed{\frac{9}{2}\tan(2t) + \frac{7}{4}\tan^2(2t) - \frac{1}{6}\tan^3(2t) + C}$

11. $\int \frac{8-w}{4w^2+9}\,dw$ $\qquad$ $\cancel{u = 4w^2+9}$ $\quad$ $\cancel{du = 8w}$

$8\int \frac{\cancel{8}\,1}{4w^2+9}\,dw - \frac{1}{8}\int \frac{8w}{4w^2+9}\,dw$

$\frac{8}{9}\int \frac{1}{\frac{4}{9}w^2+1}\,dw$ $\qquad$ $u = 4w^2+9$, $du = 8w\,dw$

$\frac{3}{2}\cdot\frac{8}{9}\int \frac{\frac{2}{3}\cdot 1}{(\frac{2}{3}w)^2+1}\,dw - \frac{1}{8}\int \frac{1}{u}\,du$

$u = \frac{2}{3}w$

$du = \frac{2}{3}\,dw$

$\frac{4}{3}\int \frac{1}{u^2+1}\,du - \frac{1}{8}\int \frac{1}{u}\,du$

$\frac{4}{3}\tan^{-1}(u) - \frac{1}{8}\ln|u| + C$

$= \boxed{\frac{4}{3}\tan^{-1}(\frac{2}{3}w) - \frac{1}{8}\ln|4w^2+9| + C}$

12. $\int \frac{7x+2}{\sqrt{1-25x^2}}\,dx$ $\qquad$ $\cancel{u = 1-25x^2}$ $\quad$ $\cancel{du = -50x}$

$\int \frac{7x}{\sqrt{1-25x^2}}\,dx + 2\int \frac{\cancel{2}\,1}{\sqrt{1-25x^2}}$

$-\frac{7}{50}\cdot\int \frac{-50x}{\sqrt{1-25x^2}} + \frac{1}{5}\cdot 2\int \frac{.5\cdot 1}{\sqrt{1-(5x)^2}}$

$u = 1-25x^2$ $\qquad$ $u = 5x$

$du = -50x\,dx$ $\qquad$ $du = 5\,dx$

$\frac{-7}{50}\int u^{-1/2}\,du + \frac{2}{5}\int \frac{1}{\sqrt{1-u^2}}\,du$

$\frac{-7}{50}\cdot\frac{2}{1}u^{1/2} + \frac{2}{5}\sin^{-1}(u) + C$

$\boxed{\frac{-7}{25\sqrt{1-25x^2}} + \frac{2}{5}\sin^{-1}(5x) + C}$

Evaluate the Integrals

a. $\int \cos(x) - \frac{3}{x^5}\,dx$ $3x^{-5}$ $= \boxed{\sin x + \frac{3}{4}x^{-4} + C}$

b. $\int_{-3}^{4} \cos(x) - \frac{3}{x^5}\,dx$ – $\boxed{\text{no solution}}$ - not continuous at $x=0$ (in interval)

c. $\int_{1}^{4} \cos(x) - \frac{3}{x^5}\,dx = \left[\sin(4) + \frac{3}{4}(4)^{-4}\right] - \left[\sin(1) + \frac{3}{4}(1)^{-4}\right] = \boxed{\sin(4) - \sin(1) - \frac{765}{1024}}$

Evaluate each of the following integrals, if possible. If it is not possible clearly explain why it is not possible to evaluate the integral.

2. $\int_1^6 12x^3 - 9x^2 + 2\,dx$ $\quad 3x^4 - 3x^3 + 2x + C \quad [3(6)^4 - 3(6)^3 + 2(6) + C] - [3(1)^4 - 3(1)^3 + 2(1) + C] = \boxed{3250}$

3. $\int_{-2}^{1} 5z^2 - 7z + 3\,dz$ $\quad \frac{5}{3}z^3 - \frac{7}{2}z^2 + 3z + C \Big|_{-2}^{1} = \boxed{\frac{69}{2}}$

4. $\int_3^0 15w^4 - 13w^2 + w\,dw$ $\quad 3w^5 - \frac{13}{3}w^3 + \frac{1}{2}w^2 + C \Big|_3^0 = \boxed{\frac{-1233}{2}}$

5. $\int_1^4 \frac{8}{\sqrt{t}} - 12\sqrt{t^3}\,dt$ $\quad 8t^{-1/2} - 12t^{3/2}$ $\quad 16t^{1/2} - \frac{24}{5}t^{5/2} + C \Big|_1^4 = \boxed{\frac{-644}{5}}$

☆ 6. $\int_1^2 \frac{1}{7z} + \frac{\sqrt[3]{z^2}}{4} - \frac{1}{2z^3}\,dz$ $\quad \frac{1}{7}z^{-1} + \frac{1}{4}z^{2/3} - \frac{1}{2}z^{-3}$ $\quad \frac{1}{7}\ln|z| + \frac{3}{20}z^{5/3} + \frac{1}{4}z^{-2} + C \Big|_1^2 = \left[\frac{1}{7}\ln|2| + \frac{3}{20}(2)^{5/2} + \frac{1}{16}\right] - \left[\frac{1}{7}\ln|1| + \frac{3}{20} + \frac{1}{4}\right] =$
$\frac{1}{7}\ln|2| + \frac{3}{20}(2)^{5/2} - \frac{27}{80}$

7. $\int_{-2}^{4} x^6 - x^4 + \frac{1}{x^2}\,dx$ $\boxed{\text{no solution}}$. discontinuous at $x=0$

8. $\int_{-4}^{-1} x^2(3-4x)\,dx$ $\quad 3x^2 - 4x^3$ $\quad x^3 - x^4 + C \Big|_{-4}^{-1} = \boxed{318}$

9. $\int_2^1 \frac{2y^3 - 6y^2}{y^2}\,dy$ $\quad 2y - 6$ $\quad y^2 - 6y + C \Big|_2^1 = \boxed{3}$

10. $\int_0^{\frac{\pi}{2}} 7\sin(t) - 2\cos(t)\,dt$ $\quad -7\cos(t) - 2\sin(t) + C \Big|_0^{\pi/2} = [0-2] - [-7-0] = \boxed{5}$

11. $\int_0^{\pi} \sec(z)\tan(z) - 1\,dz$ $\boxed{\text{no solution}}$ $\quad \sec(z) = \frac{1}{\cos(z)} \quad \cos(z) = 0$ at $\pi/2$

12. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 2\sec^2(w) - 8\csc(w)\cot(w)\,dw = 2\tan(w) + 8\csc(w)\Big|_{\pi/6}^{\pi/3} = \left[2\sqrt{3} + 8\cdot\frac{2}{\sqrt{3}}\right] - \left[\frac{2\sqrt{3}}{3} + 8\cdot 2\right] = \boxed{2\sqrt{3} + \frac{14\sqrt{3}}{3} - 16}$

13. $\int_0^2 \mathbf{e}^x + \frac{1}{x^2+1}\,dx = e^x + \tan^{-1}(x)\Big|_0^2 = \left[e^2 + \tan^{-1}(2)\right] - \left[\overset{1}{e^0} + \overset{0}{\cancel{\tan^{-1}(0)}}\right] = \boxed{e^2 + \tan^{-1}(2) - 1}$

14. $\int_{-5}^{-2} 7\mathbf{e}^y + \frac{2}{y}\,dy = 7e^y + 2\ln|y| + c\Big|_{-5}^{-2} = \boxed{7e^{-2} + 2\ln(2) - 7e^{-5} - 2\ln(5)}$

15. $\int_0^4 f(t)\,dt$ where $f(t) = \begin{cases} 2t & t > 1 \\ 1 - 3t^2 & t \le 1 \end{cases}$

16. $\int_{-6}^{1} g(z)\,dz$ where $g(z) = \begin{cases} 2 - z & z > -2 \\ 4\mathbf{e}^z & z \le -2 \end{cases}$

17. $\int_3^6 |2x - 10|\,dx$

18. $\int_{-1}^{0} |4w + 3|\,dw$

Substitution Rule for Definite Integrals

Evaluate each of the following integrals, if possible. If it is not possible clearly explain why it is not possible to evaluate the integral.

1. $\int_0^1 3(4x + x^4)(10x^2 + x^5 - 2)^6\,dx$

2. $\int_0^{\frac{\pi}{4}} \frac{8\cos(2t)}{\sqrt{9 - 5\sin(2t)}}\,dt$

3. $\int_{\pi}^{0} \sin(z)\cos^3(z)\,dz$

4. $\int_1^4 \sqrt{w}\,\mathbf{e}^{1-\sqrt{w^3}}\,dw$

5. $\int_{-4}^{-1} \sqrt[3]{5-2y} + \dfrac{7}{5-2y}\,dy$

6. $\int_{-1}^{2} x^3 + \mathbf{e}^{\frac{1}{4}x}\,dx$

7. $\int_{\pi}^{\frac{3\pi}{2}} 6\sin(2w) - 7\cos(w)\,dw$

8. $\int_{1}^{5} \dfrac{2x^3 + x}{x^4 + x^2 + 1} - \dfrac{x}{x^2 - 4}\,dx$

9. $\int_{-2}^{0} t\sqrt{3+t^2} + \dfrac{3}{(6t-1)^2}\,dt$

10. $\int_{-2}^{1} (2-z)^3 + \sin(\pi z)\left[3 + 2\cos(\pi z)\right]^3\,dz$

Applications of Integrals

Introduction

Here are a set of practice problems for the Integrals chapter of my Calcu
viewing the pdf version of this document (as opposed to viewing it on th
contains only the problems themselves and no solutions are included in t
can be found in a number of places on the site.

END

19. If you'd like a pdf document containing the solutions go to the n
you'd like solutions for and select the download solutions link fr

20. Go to the download page for the site http://tutorial.math.lamar.ec
select the section you'd like solutions for and a link will be provi

21. If you'd like to view the solutions on the web or solutions to an individual problem you can go to the problem set web page, select the problem you want the solution for. At this point I do not provide pdf versions of individual solutions, but for a particular problem you can select "Printable View" from the "Solution Pane Options" to get a printable version.

Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section.

Here is a list of topics in this chapter that have practice problems written for them.

Average Function Value

For problems 1 & 2 determine f_{avg} for the function on the given interval.

1. $f(x) = 8x - 3 + 5\mathbf{e}^{2-x}$ on $[0,2]$

2. $f(x) = \cos(2x) - \sin\left(\frac{x}{2}\right)$ on $\left[-\frac{\pi}{2}, \pi\right]$

3. Find f_{avg} for $f(x) = 4x^2 - x + 5$ on $[-2,3]$ and determine the value(s) of c in $[-2,3]$ for which $f(c) = f_{\text{avg}}$.

Area Between Curves

1. Determine the area below $f(x) = 3 + 2x - x^2$ and above the x-axis.

2. Determine the area to the left of $g(y) = 3 - y^2$ and to the right of $x = -1$.

For problems 3 – 11 determine the area of the region bounded by the given set of curves.

3. $y = x^2 + 2$, $y = \sin(x)$, $x = -1$ and $x = 2$

4. $y = \dfrac{8}{x}$, $y = 2x$ and $x = 4$

13. $\int z^7 \left(8+3z^4\right)^8 dz$

Area Problem

For problems 1 – 3 estimate the area of the region between the function and the x-axis on the given interval using $n = 6$ and using,

(a) the right end points of the subintervals for the height of the rectangles,
(b) the left end points of the subintervals for the height of the rectangles and,
(c) the midpoints of the subintervals for the height of the rectangles.

1. $f(x) = x^3 - 2x^2 + 4$ on $[1,4]$

2. $g(x) = 4 - \sqrt{x^2+2}$ on $[-1,3]$

3. $h(x) = -x\cos\left(\frac{x}{3}\right)$ on $[0,3]$

4. Estimate the net area between $f(x) = 8x^2 - x^5 - 12$ and the x-axis on $[-2,2]$ using $n = 8$ and the midpoints of the subintervals for the height of the rectangles. Without looking at a graph of the function on the interval does it appear that more of the area is above or below the x-axis?

The Definition of the Definite Integral

For problems 1 & 2 use the definition of the definite integral to evaluate the integral. Use the right end point of each interval for x_i^*.

1. $\int_1^4 2x + 3\,dx$ $\quad x^2+3x+C \quad (4)^2+3(4)+C-[(1)^2+3(1)+C] = 24$

2. $\int_0^1 6x(x-1)\,dx$

3. Evaluate : $\displaystyle\int_4^4 \frac{\cos\left(e^{3x}+x^2\right)}{x^4+1}\,dx$

For problems 4 & 5 determine the value of the given integral given that $\int_{6}^{11} f(x)\,dx = -7$ and $\int_{6}^{11} g(x)\,dx = 24$.

4. $\int_{11}^{6} 9f(x)\,dx$

5. $\int_{6}^{11} 6g(x) - 10f(x)\,dx$

6. Determine the value of $\int_{2}^{9} f(x)\,dx$ given that $\int_{5}^{2} f(x)\,dx = 3$ and $\int_{5}^{9} f(x)\,dx = 8$.

7. Determine the value of $\int_{-4}^{20} f(x)\,dx$ given that $\int_{-4}^{0} f(x)\,dx = -2$, $\int_{31}^{0} f(x)\,dx = 19$ and $\int_{20}^{31} f(x)\,dx = -21$.

For problems 8 & 9 sketch the graph of the integrand and use the area interpretation of the definite integral to determine the value of the integral.

8. $\int_{1}^{4} 3x - 2\,dx$

9. $\int_{0}^{5} -4x\,dx$

For problems 10 – 12 differentiate each of the following integrals with respect to *x*.

10. $\int_{4}^{x} 9\cos^2\left(t^2 - 6t + 1\right) dt$

11. $\int_{7}^{\sin(6x)} \sqrt{t^2 + 4}\,dt$

12. $\int_{3x^2}^{-1} \frac{\mathbf{e}^t - 1}{t}\,dt$

Computing Definite Integrals

1. Evaluate each of the following integrals.

5. $x = 3 + y^2$, $x = 2 - y^2$, $y = 1$ and $y = -2$

6. $x = y^2 - y - 6$ and $x = 2y + 4$

7. $y = x\sqrt{x^2 + 1}$, $y = \mathbf{e}^{-\frac{1}{2}x}$, $x = -3$ and the y-axis

8. $y = 4x + 3$, $y = 6 - x - 2x^2$, $x = -4$ and $x = 2$

9. $y = \dfrac{1}{x+2}$, $y = (x+2)^2$, $x = -\dfrac{3}{2}$, $x = 1$

10. $x = y^2 + 1$, $x = 5$, $y = -3$ and $y = 3$

11. $x = \mathbf{e}^{1+2y}$, $x = \mathbf{e}^{1-y}$, $y = -2$ and $y = 1$

Volumes of Solids of Revolution / Method of Rings

For problems 1 – 8 use the method of disks/rings to determine the volume of the solid obtained by rotating the region bounded by the given curves about the given axis.

1. Rotate the region bounded by $y = \sqrt{x}$, $y = 3$ and the y-axis about the y-axis.

2. Rotate the region bounded by $y = 7 - x^2$, $x = -2$, $x = 2$ and the x-axis about the x-axis.

3. Rotate the region bounded by $x = y^2 - 6y + 10$ and $x = 5$ about the y-axis.

4. Rotate the region bounded by $y = 2x^2$ and $y = x^3$ about the x-axis.

5. Rotate the region bounded by $y = 6\mathbf{e}^{-2x}$ and $y = 6 + 4x - 2x^2$ about the line $y = -2$.

6. Rotate the region bounded by $y = 10 - 6x + x^2$, $y = -10 + 6x - x^2$, $x = 1$ and $x = 5$ about the line $y = 8$.

7. Rotate the region bounded by $x = y^2 - 4$ and $x = 6 - 3y$ about the line $x = 24$.

8. Rotate the region bounded by $y = 2x + 1$, $x = 4$ and $y = 3$ about the line $x = -4$.

Volumes of Solids of Revolution / Method of Cylinders

For problems 1 – 8 use the method of cylinders to determine the volume of the solid obtained by rotating the region bounded by the given curves about the given axis.

1. Rotate the region bounded by $x = (y-2)^2$, the x-axis and the y-axis about the x-axis.

2. Rotate the region bounded by $y = \frac{1}{x}$, $x = \frac{1}{2}$, $x = 4$ and the x-axis about the y-axis.

3. Rotate the region bounded by $y = 4x$ and $y = x^3$ about the y-axis. For this problem assume that $x \geq 0$.

4. Rotate the region bounded by $y = 4x$ and $y = x^3$ about the x-axis. For this problem assume that $x \geq 0$.

5. Rotate the region bounded by $y = 2x+1$, $y = 3$ and $x = 4$ about the line $y = 10$.

6. Rotate the region bounded by $x = y^2 - 4$ and $x = 6 - 3y$ about the line $y = -8$.

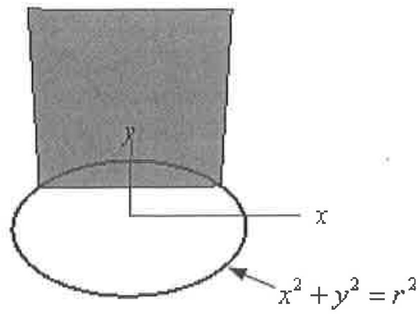
7. Rotate the region bounded by $y = x^2 - 6x + 9$ and $y = -x^2 + 6x - 1$ about the line $x = 8$.

8. Rotate the region bounded by $y = \dfrac{\mathbf{e}^{\frac{1}{3}x}}{x+2}$, $y = 5 - \frac{1}{4}x$, $x = -1$ and $x = 6$ about the line $x = -2$.

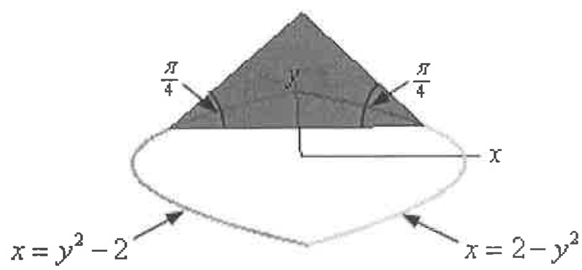
More Volume Problems

1. Find the volume of a pyramid of height h whose base is an equilateral triangle of length L.

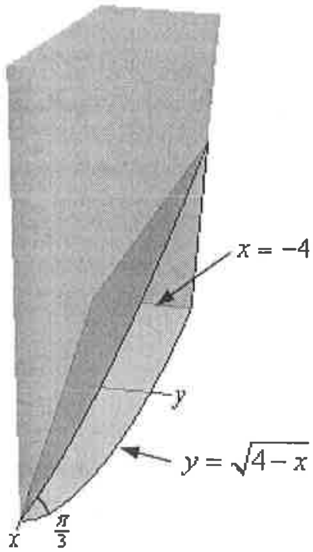
2. Find the volume of the solid whose base is a disk of radius r and whose cross-sections are squares. See figure below to see a sketch of the cross-sections.

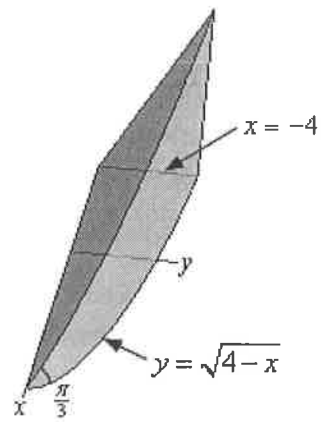


3. Find the volume of the solid whose base is the region bounded by $x = 2 - y^2$ and $x = y^2 - 2$ and whose cross-sections are isosceles triangles with the base perpendicular to the y-axis and the angle between the base and the two sides of equal length is $\frac{\pi}{4}$. See figure below to see a sketch of the cross-sections.



4. Find the volume of a wedge cut out of a "cylinder" whose base is the region bounded by $y = \sqrt{4 - x}$, $x = -4$ and the x-axis. The angle between the top and bottom of the wedge is $\frac{\pi}{3}$. See the figure below for a sketch of the "cylinder" and the wedge (the positive x-axis and positive y-axis are shown in the sketch – they are just in a different orientation).





Work

1. A force of $F(x) = x^2 - \cos(3x) + 2$, x is in meters, acts on an object. What is the work required to move the object from $x = 3$ to $x = 7$?

2. A spring has a natural length of 18 inches and a force of 20 lbs is required to stretch and hold the spring to a length of 24 inches. What is the work required to stretch the spring from a length of 21 inches to a length of 26 inches?

3. A cable that weighs ½ kg/meter is lifting a load of 150 kg that is initially at the bottom of a 50 meter shaft. How much work is required to lift the load ¼ of the way up the shaft?

4. A tank of water is 15 feet long and has a cross section in the shape of an equilateral triangle with sides 2 feet long (point of the triangle points directly down). The tank is filled with water to a depth of 9 inches. Determine the amount of work needed to pump all of the water to the top of the tank. Assume that the density of water is 62 lb/ft^3.