Calculus

Name Key

Unit 4 - Review

Date

Evaluate each indefinite integral.

1) $\int (-30x^5 + 10x^4 + 8x)\, dx$

$\boxed{-5x^6 + 2x^5 + 4x^2 + C}$

2) $\int \left(7\sqrt[5]{x^2} + \frac{8\sqrt[3]{x}}{3} + \frac{25\sqrt[4]{x}}{4}\right) dx$

$\int 7x^{2/5} + \frac{8}{3}x^{1/3} + \frac{25}{4}x^{1/4}\, dx$

$7 \cdot \frac{5}{7}x^{7/5} + \frac{8}{3}\cdot\frac{3}{4}x^{4/3} + \frac{25}{4}\cdot\frac{4}{5}x^{5/4} + C$

$\boxed{5x^{7/5} + 2x^{4/3} + 5x^{5/4} + C}$

3) $\int (-6\sqrt[5]{x} - 2 + x^{-2})\, dx$

$\int -6x^{1/5} - 2 + x^{-2}\, dx$

$-6 \cdot \frac{5}{6}x^{6/5} - 2x - 1x^{-1} + C$

$\boxed{-5x^{6/5} - 2x - 1x^{-1} + C}$

4) $\int \left(15x^2 + \frac{3x^{\frac{1}{2}}}{2} + 6x^{-3}\right) dx$

$\int 15x^2 + \frac{3}{2}x^{1/2} + 6x^{-3}\, dx$

$15 \cdot \frac{1}{3}x^3 + \frac{3}{2}\cdot\frac{2}{3}x^{3/2} + 6 \cdot \frac{1}{-2}x^{-2} + C$

$\boxed{5x^3 + x^{3/2} - 3x^{-2} + C}$

5) $\int 5x^{-1}\, dx$ 4

$5\int x^{-1}\, dx$

$\boxed{5\ln|x| + C}$

6) $\int 2e^x\, dx$ 4

$2\int e^x\, dx$

$\boxed{2e^x + C}$

7) $\int -4\sec x \cdot \tan x \, dx$

$-4\int \sec x \tan x \, dx$

$\boxed{-4\sec x + C}$

8) $\int 5\cos x \, dx$

$5\int \cos x \, dx$

$\boxed{5\sin x + C}$

9) $\int 2\cot x \, dx$

$2\int \frac{\cos x}{\sin x} dx$

$u = \sin x$

$du = \cos x \, dx$

$2\int \frac{1}{u} du$

$2\ln|u| + C$

$\boxed{2\ln|\sin(x)| + C}$

10) $\int 15x^2(5x^3 - 1)^5 \, dx$

$u = 5x^3 - 1$

$du = 15x^2 dx$

$\int u^5 du$

$\frac{1}{6}u^6 + C = \boxed{\frac{1}{6}(5x^3-1)^6 + C}$

11) $5 \cdot \int (4x^5 - 1)^{-3} \cdot \frac{1}{5} \cdot 100x^4 \, dx$

$u = 4x^5 - 1$

$du = 20x^4 dx$

$5\int u^{-3} du$

$5 \cdot \frac{-1}{2} u^{-2} + C = \boxed{\frac{-5}{2}(4x^5-1)^{-2} + C}$

12) $5 \cdot \int \frac{30x \cdot \frac{1}{5}}{(3x^2 - 2)^5} dx$

$u = 3x^2 - 2$

$du = 6x \, dx$

$5\int \frac{1}{u^5} du = 5\int u^{-5} du$

$5 \cdot \frac{-1}{4} u^{-4} + C = \boxed{\frac{-5}{4}(3x^2-2)^{-4} + C}$

13) $\frac{1}{2} \int \frac{(-4 + \ln 2x)^3}{x} \cdot 2 \, dx$

$u = -4 + \ln(2x)$

$du = \frac{1}{x} \cdot 2 = \frac{2}{x} dx$ chain rule

$\frac{1}{2}\int u^3 du$

$\frac{1}{2} \cdot \frac{1}{4} u^4 + C = \boxed{\frac{1}{8}(-4 + \ln(2x))^4 + C}$

14) $\int -\cos(-x) \cdot \sin^4(-x) dx$

$\int -\cos(-x)(\sin(-x))^4 dx$

$u = \sin(-x)$

$du = \cos(-x) \cdot -1 \, dx$ chain rule

$= -\cos(-x) \, dx$

$\int u^4 du$

$\frac{1}{5}u^5 + C = \boxed{\frac{1}{5}\sin^5(-x) + C}$

15) $\int \sin(-x)\cos^5 -x\,dx$

$\int \sin(-x)[\cos(-x)]^5\,dx$

$u=\cos(-x)$ chain rule

$du=-\sin(-x)\cdot -1\,dx$

$= \sin(-x)\,dx$

$\int u^5\,du$

$\frac{1}{6}u^6+c=\boxed{\frac{1}{6}\cos^6(-x)+c}$

16) $3.\int^{\cdot\frac{1}{3}} -6\csc(2x)\cdot\cot(2x)\overbrace{\csc^3(2x)\,dx}^{du}$

$\int -6\csc(2x)\cot(2x)[\csc(2x)]^3\,dx$

$u=\csc(2x)$ chain rule

$du=-\csc x\cot x\cdot 2\,dx$

$=-2\csc x\cot x\,dx$

$3\int u^3\,du$

$3\cdot\frac{1}{4}u^4+c=\boxed{\frac{3}{4}\csc^4(2x)+c}$

17) $4\int^{\cdot\frac{1}{4}} -20\cos(-5x)\cdot\sin^4(-5x)dx$

$\int -20\cos(-5x)[\sin(-5x)]^4\,dx$

$u=\sin(-5x)$ chain rule

$du=\cos(-5x)\cdot -5\,dx$

$=-5\cos(-5x)\,dx$

$4\int u^4\,du$

$4\cdot\frac{1}{5}u^5+C=\boxed{\frac{4}{5}\sin^5(-5x)+C}$

18) $\int (e^{4x}-4)^3\cdot 4e^{4x}\,dx$

$u=e^{4x}-4$

$du=4e^{4x}\,dx$

$\int u^3\,du$

$\frac{1}{4}u^4+c=\boxed{\frac{1}{4}(e^{4x}-4)^4+c}$

19) $4\int (2x^2+5)^3\cdot \overset{\cdot\frac{1}{4}}{16x}\,dx$

$u=2x^2+5$

$du=4x\,dx$

$4\int u^3\,du$

$4\cdot\frac{1}{4}u^4+c=\boxed{(2x^2+5)^4+c}$

20) $\int \frac{4}{4+4x^2}\,dx$

$\int \frac{\cancel{4}}{\cancel{4}(1+x^2)}\,dx$

$\int \frac{1}{1+x^2}\,dx$

$\boxed{\tan^{-1}(x)+C}$

~~Evaluate each definite integral.~~

~~20)~~ $\cancel{\int_5^{10} x\,dx}$

21) $\int \frac{20x^4}{4+16x^{10}}\,dx$

$\int \frac{\overset{5}{\cancel{20}}x^4}{\cancel{4}(1+4x^{10})}\,dx$

$\frac{1}{2}\cdot\int \frac{2\cdot 5x^4}{1+(2x^5)^2}\,dx$

$u=2x^5$

$du=10x^4\,dx$

$\frac{1}{2}\int \frac{1}{1+u^2}\,du$

$\boxed{\frac{1}{2}\tan^{-1}(2x^5)+c}$

22) $\int \frac{4x}{\sqrt{9-4x^4}}\,dx$

$\int \frac{4x}{\sqrt{9(1-\frac{4}{9}x^4)}}\,dx$

$\int \frac{4x}{3\sqrt{1-(\frac{2}{3}x^2)^2}}$

$u=\frac{2}{3}x^2$

$du=\frac{4}{3}x\,dx$

$\int \frac{1}{\sqrt{1-u^2}}\,du$

$\sin^{-1}(u)+c$

$=$

$\boxed{\sin^{-1}(\frac{2}{3}x^2)+c}$

21) $\int_{-2}^{2} 5x^{\frac{1}{3}}\, dx$

22) $\int_{-3}^{-2} \frac{1}{x^2}\, dx$

23) $\int_{-2}^{1} e^x\, dx$

24) $\int_{2}^{4} -\frac{3}{x}\, dx$

25) $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \cos x\, dx$

26) $\int_{-\frac{\pi}{6}}^{\frac{2\pi}{3}} 2\sin x\, dx$

Evaluate each definite integral.

23) $\int_{-5}^{0} x\,dx$ $\frac{1}{2}x^2 + c \Big|_{-5}^{0}$

$0 - \left(\frac{1}{2}(-5)^2\right) = \boxed{-\frac{25}{2}}$

24) $\int_{-2}^{2} 5x^{\frac{1}{3}}\,dx$ $\frac{5.3}{4}x^{4/3} + C \Big|_{-2}^{2}$

$\frac{15}{4}(2)^{4/3} - \frac{15}{4}(-2)^{4/3} = \boxed{0}$

25) $\int_{-3}^{-2} \frac{1}{x^2}\,dx$ $\int_{-3}^{-2} x^{-2}dx = -x^{-1} + c \Big|_{-3}^{-2}$

$-(-2)^{-1} + (+(-3)^{-1})$

$+\frac{1}{+2} + \frac{1}{-3} = \boxed{\frac{1}{6}}$

26) $\int_{-2}^{1} e^x\,dx$ $e^x + c \Big|_{-2}^{1}$

$\boxed{e^1 - e^{-2}}$

27) $\int_{2}^{4} -\frac{3}{x}\,dx$ $\int_2^4 -3x^{-1}dx = -3\ln|x| \Big|_2^4$

$\boxed{-3\ln(4) + 3\ln(2)}$

28) $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \cos x\,dx = \sin x + c \Big|_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$

$\sin\left(\frac{3\pi}{4}\right) - \sin\left(\frac{\pi}{4}\right)$

$\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = \boxed{0}$

29) $\int_{-\frac{\pi}{6}}^{\frac{2\pi}{3}} 2\sin x\,dx = -2\cos(x) + c \Big|_{-\pi/6}^{2\pi/3}$

$-2\cos\left(\frac{2\pi}{3}\right) + +2\cos\left(-\frac{\pi}{6}\right)$

$-2\left(-\frac{1}{2}\right) + +2\left(\frac{\sqrt{3}}{2}\right) = \boxed{1 + \sqrt{3}}$