Calculus – Unit 1 Review Problems Name: Key

Functions and Exponents:

Simplify each completely.
Leave no negative or fractional exponents.

1. $(4h^3)^2(-2g^3h)^3 =$

$16h^6 \cdot -8g^9h^3$

$-128h^9g^9$

$\boxed{-128g^9h^9}$

2. $\left(\frac{-24t^6}{8t^3}\right)^5 =$

$(-3t^3)^5$

$\boxed{-243t^{15}}$

3. $\frac{z^{x+1}}{z^x} =$

subtract exponents

$z^{(x+1)-x}$

z^1

$\boxed{z}$

4. $5\sqrt[3]{3} + 2\sqrt[3]{24} - \sqrt[3]{81} =$

$5\sqrt[3]{3} + 2\sqrt[3]{8\cdot3} - \sqrt[3]{3\cdot27}$

$5\sqrt[3]{3} + 4\sqrt[3]{3} - 3\sqrt[3]{3}$

$\boxed{6\sqrt[3]{3}}$

5. $\sqrt[3]{54a^4b^{10}} =$

$\sqrt[3]{27\cdot2a^3ab^9b}$

$\boxed{3ab^3\sqrt[3]{2ab}}$

6. $\frac{\sqrt[5]{64x^2y^6}}{\sqrt[5]{2x^7y}} =$

$\sqrt[5]{\frac{32y^5}{x^5}}$

$\boxed{\frac{2y}{x}}$

Find (f ∘ g)(x) and (g ∘ f)(x).

7. $f(x) = \frac{x^2-1}{x-3}$ $g(x) = 3x + 5$

fog: $\frac{(3x+5)^2-1}{(3x+5)-3}$ ← foil

$\frac{9x^2+30x+25-1}{3x+2}$

$\boxed{\frac{9x^2+30x+24}{3x+2}}$

$\boxed{\frac{3(3x+4)(x+2)}{(3x+2)}}$

gof: $3\left(\frac{x^2-1}{x-3}\right)+5$

$\frac{3x^2-3}{x-3}+5$

$\frac{3x^2-3}{x-3} + \frac{5(x-3)}{x-3}$

$\boxed{\frac{3x^2+5x-18}{x-3}}$

8. $f(x) = x^2$ $g(x) = x^3 + 1$

fog: $(x^3+1)^2$

foil

$\boxed{x^6+2x^3+1}$

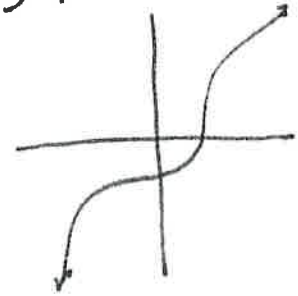
gof: $(x^2)^3+1$

$\boxed{x^6+1}$

Determine whether the true inverse exists.
If it does, find the equation of the inverse.
If it does not, explain why it does not exist.

9. $f(x) = \sqrt[3]{(x^5 - 1)}$

graph:



$x = \sqrt[3]{y^5 - 1}$

$x^3 = y^5 - 1$

$x^3 + 1 = y^5$

$y = \sqrt[5]{x^3 + 1}$

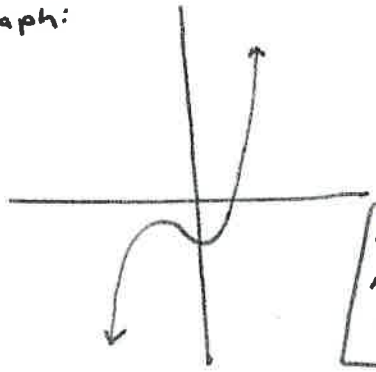
$f^{-1}(x) = \sqrt[5]{x^3 + 1}$

10. $f(x) = x^3 + 2x^2 - 5$

$x = y^3 + 2y^2 - 5$

multiple differing y exponents

graph:



original function

no inverse - does not pass horizontal line test

Trigonometry:

Solve each equation.
If your unit circle can be used for an exact answer, you should avoid using a calculator.

11. $\tan^2\theta - 2\tan\theta = 0$

$\tan\theta(\tan\theta - 2) = 0$

$\tan\theta = 0$

$\frac{\sin\theta}{\cos\theta} = 0$

where is sin 0 on unit circle?

$\theta = 0, \pi$

$\theta = 0 + 2\pi n$

$\theta = \pi + 2\pi n$

$\tan\theta - 2 = 0$

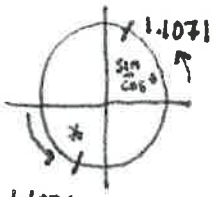
$\tan\theta = 2$

$\frac{\sin\theta}{\cos\theta} = 2$

not on circle

$\theta = \tan^{-1}(2)$

$\theta = 1.1071$



$\pi + 1.1071$

4.2487

$\theta = 1.1071 + 2\pi n$

$\theta = 4.2487 + 2\pi n$

12. $2\cos(6x) = \sqrt{12}$ on $[0, \pi]$

$\cos(6x) = \frac{2\sqrt{3}}{2}$

$\cos(6x) = \sqrt{3}$

not on unit circle

$\cos^{-1}\sqrt{3} = 6x$

Δ

note: $\sqrt{3} \approx 1.7321\ldots$ greater than 1

Δ - no solution

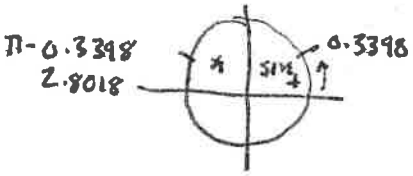
13. $18 + 2\csc(x/3) = 24$ on $[0, 5]$

$\csc(x/3) = 3$

$\sin(x/3) = 1/3$ not on unit circle

$\sin^{-1}(1/3) = x/3$

$x/3 = 0.3398$



$x/3 = 0.3398 + 2\pi n$ $\quad x/3 = 2.8018 + 2\pi n$

$x = 1.0194 + 6\pi n$ $\quad x = 8.4054 + 6\pi n$

$n = -1$	-17.8302	-10.4442
$n = 0$	1.0194	8.4054
$n = 1$	19.8690	27.2550

$x = 1.0194$

14. $4x\sec(7x) = -21x$

$4x\sec(7x) + 21x = 0$

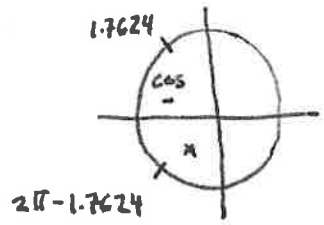
$x(4\sec(7x) + 21) = 0$

$x = 0$ $\quad 4\sec(7x) + 21 = 0$

$\sec(7x) = -21/4$

$\cos(7x) = -4/21$ not on unit circle

$\cos^{-1}(-4/21) = 1.7624$



$7x = 1.7624 + 2\pi n$ $\quad 7x = 4.5207 + 2\pi n$

$x = 0 + 2\pi n$ $\quad x = 0.2518 + 2\pi n/7$ $\quad x = 0.6458 + 2\pi n/7$

Domain and Range of Functions

Find the domain.

15. $h(y) = \dfrac{\sqrt{2y+9}}{\sqrt{2-y}}$

must be ≥ 0 $\quad$ must be > 0

$2y + 9 \geq 0$ AND $2 - y > 0$

$y \geq -9/2$ $\quad y < 2$

$y \geq -9/2$

$y < 2$

$-9/2 \quad 2$

$[-9/2, 2)$

16. $g(t) = \sqrt{6 + t - t^2}$

$\sqrt{-t + t + 6}$

$\sqrt{-(t - t - 6)}$

$\sqrt{-(t-3)(t+2)}$

product of 3 values

first is always negative

$t-3$ pos AND $t+2$ neg $\quad$ $t-3$ neg AND $t+2$ pos

$-2 \quad 3$

Δ

$[-2, 3]$

$[-2, 3]$

$6 + t - t^2 \geq 0$

$t^2 - t - 6 \leq 0$

$(t-3)(t+2) \leq 0$

$t \leq 3 \quad t \geq -2$

Exponential and Logarithmic Functions:

Solve for x.

17. $\log_{125} 5 = x$

$125^x = 5$

$(5^3)^x = 5^1$

$3x = 1$

$x = 1/3$

18. $\log_x \frac{1}{2} = -1$

$x^{-1} = 1/2$

$x = 2$

19. $\log_4 8 = x$

$4^x = 8$

$(2^2)^x = 2^3$

$2x = 3$

$x = 3/2$

Solve each equation.

20. $\ln x + \ln(x - 4) = \ln 5$

$\ln[x(x-4)] = \ln 5$

$x^2 - 4x = 5$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x = 5$ $x = -1$ not a soln.

21. $e^{x-1} - 4 = 4$

$e^{x-1} = 8$

$\ln e^{x-1} = \ln 8$

$x - 1 = \ln 8$

$x = \ln 8 + 1$

challenge

22. $\ln(x^2 - 11) = \ln(9) + \ln(x)$

$\ln(x^2 - 11) = \ln(9x)$

$x^2 - 11 = 9x$

$x^2 - 9x - 11 = 0$

use quad. formula: $\frac{9 \pm \sqrt{81 + 44}}{2}$

$x = \frac{9 + 5\sqrt{5}}{2}$ $x = \frac{9 - 5\sqrt{5}}{2}$ not a soln.

23. $e^{(x+2)} = e^{3x}$

$x + 2 = 3x$

$2 = 2x$

$x = 1$

24. $(1/4)^{2x-3} = 16$

$(4^{-1})^{2x-3} = 4^2$

$-2x + 3 = 2$

$-2x = -1$

$x = 1/2$

25. $\log(7x + 2) = 2$

$10^{\log(7x+2)} = 10^2$

$7x + 2 = 100$

$7x = 98$

$x = 14$